SOUND

FOR ADVANCED STUDENTS

BY THE SAME AUTHOR.

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SOUND

FOR ADVANCED STUDENTS

BY

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SECOND EDITION

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PREFACE.

This treatise is intended for students preparing for the Degree Examinations (Pass and Honours) of the Indian Universities. To suit all classes of students the elementary portions of the subject have been incorporated, and analytical as well as graphical expositions have been included as far as practicable.

The author is indebted for assistance to almost all the important treatises on Sound, and his thanks are due to Prof. J. N. Sen M.A., of the City College, Calcutta, for the revision of some of the proofs and many valuable suggestions.

PREFACE TO THE REVISED EDITION.

In this edition some additional matter has been introduced, and such misprints as were detected have been corrected. Some of the blocks of apparatus have been modified.

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SOUND.

CHAPTER I.

INTRODUCTORY.

1. **Sound.** We hear a church bell ringing at a distance. It will be seen later that when the bell is struck it vibrates, and its vibratory motion is transmitted through air in the form of waves to the ear, causing the perception of sound. Sound is thus a kind of sensation received through the ear, and produced by the vibratory motion of material bodies, when this motion is transmitted to the ear through some elastic medium. It is commonly used either to denote the sensation perceived, or the external disturbance which arouses that sensation when it reaches the ear.

2. **Acoustics** or the science of sound is that branch of Physics which deals with the nature, phenomena and laws of sounds.

3. **Nerve and senses.** The various nerves of our body have their centres at different parts of the brain, which are the seats of different kinds of sensation. When stimulus is applied to some part of our body, the nerves there transmit the message to the brain in the form of impulses and arouse some kind of sensation. Thus when a finger is hurt the message of the injury is conveyed by the *sensor* nerves and pain is perceived; when a flower is held near the nose, the *olfactory* nerves being excited arouse the

sense of smell. In the same way impulses sent along the *taste* nerves awake the sense of taste. When the image of an object falls on the retina the *optical* nerves are stimulated to transmit impulses to the brain to produce vision. In like manner, sound waves enter the ear and cause impulses to be transmitted along the *auditory* nerve to its centre, where as stated by Tyndall, the physical becomes the psychical, mechanical vibrations giving birth to the consciousness of sound.

4. **Ultimate cause or origin of sound.** Whenever we hear any sound, if we trace it up, we always find as its source a material body in a state of vibration. The vibrations may be very rapid and the amplitude very small, so that we may not percieve the vibratory motion of the source with our naked eye. We may feel it however easily if we touch the source.

Exp. A metal bowl is fixed on a stand and a glass bead is suspended by its side, so that it just touches the edge of the bowl. If now a fiddle bow is drawn against the edge of the bowl a sharp metallic sound is given out and the glass bead is set into motion. If the bowl be partially filled with water, we find ripples formed on the water surface, showing that the bowl, while emitting sound, is in a state of vibration.

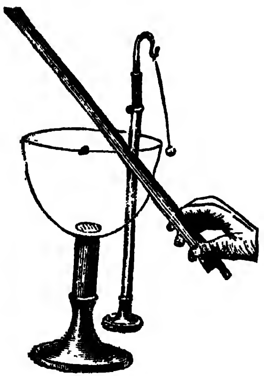


FIG. 1.

Exp. Fig 2. shows a horizontal metal plate mounted on a stand with sand spread on its upper surface. When the plate is bowed against its edge, it gives out a sharp note and the sand particles are thrown into vibration.

Fig.

Exp. Let a small plane mirror be attached to the string of a violin, and let light from some source falling on the mirror get reflected from it and fall on the wall. When the string is bowed, it gives out sound and the spot of light on the wall vibrates.

5. Sound is not propagated through vacuo.

Exp. An electric bell is placed inside a glass jar, the air inside which can be exhausted by an air-pump. When the inside of the jar is in communication with the outside air, the sound given out by the bell is distinctly heard even from a distance. The air inside the jar is now gradually pumped out and it is found that though the bell works all right the intensity of the sound falls off

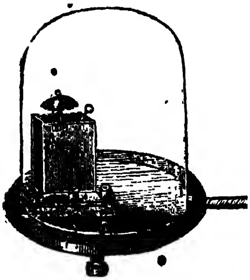


Fig. 3.

gradually, and when the air is completely exhausted no sound is perceived. Air is now gradually introduced inside the jar by turning a stopcock, and the sound begins gradually to rise in intensity, and when the pressure inside and outside the jar is the same the original intensity of sound is restored.

6. Principal requisites for the production, propagation and perception of sound. We thus see that to perceive sound there must be

(i) a vibrating source,

(ii) a transmitting medium,

(iii) a healthy organ of hearing.

Conditions to be satisfied by the source.

(*a*) The frequency of vibration (Art. 9) of the source must fall within a limited range, for it is found that when the frequency is below a certain value the human ear fails to perceive anything. Above this limit the sound becomes perceptible, and it becomes more and more acute with increasing frequency. But when the frequency exceeds a certain limiting value the sound again becomes inaudible though the source continues vibrating. Human ears, therefore, are sensitive only to this range of vibration. Above and bellow this range we are practically deaf. It varies within wide limits for different individuals who have different capacities for the preception of sounds. The lower limit is not far from 30 vibrations per second and the upper one about 40,000. Birds and animals whose ears are sensitive beyond this range, can hear sound when it ceases to be audible to us. (*b*) Again in order that sound may be perceived the disturbance must be carried from the source to the ear, otherwise there will be no effect. The mode of vibration of the source, therefore, should be such as

not to produce merely a local disturbance of the medium, but cause its continuous propagation in all directions.

The medium. The medium must be *continuous* and *elastic.* It may be a solid, a liquid or a gas.

By substituting other gases for air in the bell-jar experiment it is found that sound is transmitted through these gases.

The ticks of a watch held against one side of a table are distinctly heard by applying ear to the other side of it.

Stethoscope gives a familiar example of transmission of sound through a solid.

The sound of an approaching train can be heard by applying ear to the rails long before the sound is heard in air.

A diver inside water distinctly hears sounds produced either in water or in air. Fishes are provided with auditory organs and can hear sound.

7. **Sound takes time to travel.** The report of a gun fired at a distance is heard after the flash is seen. The velocity of light being about 186,000 miles per second, the time which elapses from the instant when the flash is seen to the instant when the report is heard, is practically the time taken by sound to travel from the source to the observer.

8. **Modes of propagation of a disturbance.** There are in general three different modes of propagation of a disturbance through a medium.

(*a*) A disturbance may be carried by moving bodies like the bullets shot off from a gun. The vibrating source

may be assumed to shoot out particles in all directions, which on entering the cavity of the ear and striking the tympanum may ultimately excite the sensation of sound.

(*b*) The transmission of a disturbance may be effected by the displacement of the medium as a whole. Let there be a ball in contact with one end of a rod. Apply a stroke at the other end towards the ball. The rod moves as a whole and sets the ball into motion.

(*c*) A disturbance may be transferred from particle to particle of a medium, each particle moving in its turn a little and coming to rest after the disturbance has passed it.

Examples. 1. Imagine a row of bricks set upright and arranged regularly, the distance between them being less than their length. On striking the first brick towards the second it falls and strikes the second brick, which in its turn falls while the first one comes to rest. The second brick makes the third brick fall while itself comes to rest and so on. Thus the disturbance set up at the first brick moves on from brick to brick.

2. *Water waves.* Imagine a water surface at rest. Agitate the water somewhere by alternately dipping and raising a small rod. Ripples will be formed which will propagate onwards uniformly in all directions in the form of circles of alternate crests and troughs of gradually increasing radii. Float a piece of paper on water at a distance from the source of disturbance. The paper originally at rest rises and falls as the waves advance through it, and again comes to rest when the waves have passed away. Like the paper the water particles through which the waves move are not

bodily carried by the waves, but they rise and fall about their positions of rest with the passing crests and troughs. The actual motion of a particle, however is round and round in a closed loop in a vertical plane, and this up and down motion is transferred from particle to particle, causing propagation of the waves onwards.

3. *Rope waves.* Imagine a thick rope suspended from the ceiling of the lecture theatre. Move the free end of the rope a little to and fro at right angles to its length. A disturbance in the form of a wave advances along the rope upwards. The particles of the rope are not bodily carried up by the waves but simply move a little to and fro about their positions of rest when the waves pass through them.

9. **Mechanism of sound propagation.** We now proceed to investigate by which of these three methods is sound transmitted from the source to the ear. The fact that sound is not transmitted through vacuo makes the first hypothesis untenable. That sound is not propagated by the second method is also evident from the following facts.

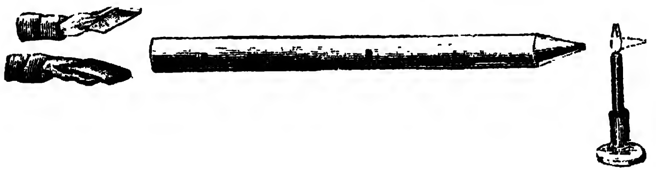


Fig. 4.

Exp A long wide tube open at one end and ending in a funnel with a nozzle at the other is placed on the lecture table. A steady candle flame is held just in front of the nozzle. The space inside the other end of the tube is filled with thick fumes. On producing a report by clapping two blocks of wood near the open end the flame is found to be

struck after a short time, though no fumes come out through the nozzle. This fact clearly leads us to infer that when sound moves from one point to another in a medium, the medium as a whole remains at rest. Evidently then transmission of sound must be due to the third method. Each particle of the medium moves a little to and fro about its position of rest, and this motion is transferred from particle to particle onwards with the velocity of sound. To follow clearly how sound is propagated let us take an open tube provided with an air-tight piston at one end. The air inside the tube may be divided in imagination into a number of very thin layers.

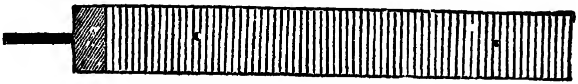


FIG. 5.

Let the piston be pushed a little inside very quickly. The layer of air just in front of the piston gets compressed The compressed layer of air tries to expand and in doing so reacts on the second layer, which in its turn gets compressed while the first layer regains its initial pressure. The second layer similarly reacts on the third layer which gets compressed while the second layer regains its initial pressure and so on. The disturbance which is a state of compression in this case, thus moves on from layer to layer with a definite velocity. If now the piston be pulled out a little very quickly, air from the first layer will rush into the space moved through by the piston and will get rarefied. This layer tries to regain its initial pressure, and air from the second layer rushes into the first layer. The first layer thus regains its initial pressure while the second layer gets rarefied. Air from the third layer now enters the second layer and so on. Thus the distur-

bance which is a state of rarefaction in this case moves on from layer to layer just like the state of compression previously considered. If the piston be moved in and out alternately in quick succession, a train of waves will be produced consisting of alternate compressions and rarefactions, which will move onwards with the velocity of sound.

The same thing will happen if a vibrating plate, for example, be held in place of the piston near one end of the tube.

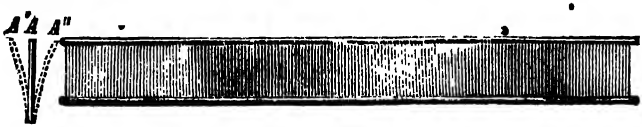


FIG. 6.

The movement of the plate from its extreme backward position A' to its extreme forward position A'' may be divided in imagination into a number of successive parts, during each of which the layer of air in front of it will get compressed. Similarly during such successive parts of its backward movement from A'' to A' the layer of air in front of the retreating plate will get rarefied. * The velocity of the plate is maximum when it passes its position of rest A, whole of its energy being kinetic in this position. Again the velocity of the plate is temporarily zero at A'' and A', whole of its energy being potential in these positions. In intermediate positions the velocity is intermediate between the maximum and zero, the energy being partly potential and partly kinetic. † The amount of compression or rarefaction being proportional to the velocity

* One complete to and fro motion of the plate constitutes one complete vibration and the number of vibrations per second is called the *frequency* of its vibration (Art 25)

† Art 93 General Physics

of the vibrating plate, there will be maximum compression when the plate passes A during its forward movement from A' to A''. Similarly there will maximum rarefaction when the plate passes A during its backward movement. At A' and A'' there is no compression or rarefaction and in other positions the effects are intermediate between the maximum and zero. The particles move along the line of advance of the waves during compression, and in the opposite direction during rarefaction. Thus when a train of sound waves consisting of alternate compressions and rarefactions, moves through a medium, the particles through which it advances simply move to and fro about their positions of rest just like the source. When this train reaches the ear, the tympanum is thrown into similar state of vibration which finally excites the auditory nerve and produces sound.

Graphical representation. In the figure 1, 2, 3, 4 etc. represent the successive positions of the plate during its vibration. 1 and 9 correspond to A', 3 and 7 to A and 5 to A'' in Fig. 6. The energy of motion of the vibrating plate is communicated to the adjacent particles which thus take up the motion of the plate, and in their turn transfer their energy to the particles in front of them and so on. The disturbance set up by the plate in moving from 1 to 2 (fig. 7), moves on, being closely followed by the disturbances set up by the plate during the successive stages of its movement from 2 to 3, 3 to 4 etc. In Fig. 8 (i) undisturbed particles of the medium are represented by a row of equidistant points. In (ii) the numbers represent the advance of the successive stages of the disturbance corresponding to the different positions of

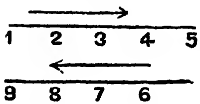


Fig 7

the plate in Fig. 7. In (iii) the actual displacements of the particles at the same instant are shown. The displacement of the plate at 1 is maximum and towards the left, and its velocity is zero. In (iv) the displacements of the particles at the same instant are represented in the form of a curve, with the convention that displacements to the left are represented by upward ordinates and to the right by downward ordinates. In (v) the velocities of the particles at the same instant are represented by the size of the arrowheads, and in the corresponding velocity curve (vi) velocities to the right are represented by upward ordinates and velocities to

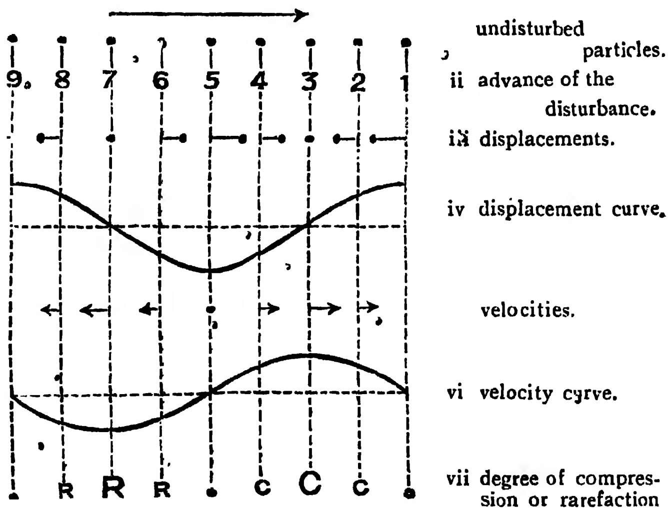


FIG. 8.

the left by downward ordinates The displacement at 3 is zero and the velocity is maximum and towards the right. The velocity and displacement corresponding to other positions are represented by the curves iv and vi. In (vii) the size of *C* or *R*

is proportional to the degree of compression or rarefaction, and the absence of C or R at cartain points shows that the pressure there is normal and therefore the change in density nil.

10. **Classification of sounds**—Sounds are generally of two classes, *musial sounds* and *noises*. Those which produce pleasant sensation are called musical sounds. We like smoothness and regularity in sound, and musical sounds are, therefore, charaterised by their smooth and regular flow. The ultimate cause of sound being the vibratory motion of its source, musical sounds must be emitted by bodies in regular or periodic vibrations. The principal feature of a musical sound is thus its periodicity. Sounds other than musical sounds are called *non-musical sounds* or *noises* The chief feature of a noise as distinguished from a musical sound is its want of regularity or smoothness. These are extreme cases however. In practice we find that musical sounds are seldom free from some irregularities, the vibrations of the source not being maintained with perfect periodicity. On the other hand even in noises some phases of the vibratory motion are repeated at regular intervals, thus producing some regular wave motion in the irregular disturbance. The two classes of sound, therefore, shade into one another and it is not possible to draw a sharp line of demarcation between them.

11 **Principal features of musical sounds.** The distinguishing features of musical sounds are their *pitch, intensity and character.*

Pitch is that feature of musical sounds by means of which an acute note is distinguished from a grave one. It depends on the rapidity of vibration of the air in contact

with the tympanum and therefore on the rapidity of vibration of the source. The physical cause of pitch, therefore, is the frequency of vibration of the source, higher frequency producing higher pitch In Physics pitch represents the frequency, but in music it assigns to the sound its position in a fixed series of sounds forming the musical scale.

Intensity of any sound, either musical sound or noise, may represent either a quantity of energy or a degree of sensation.

Intensity regarded as a quantity of energy. When there is flow of any kind of energy the intensity at a point is given by the quantity of energy flowing per unit time per unit area about that point at right angles to the direction of flow. In the case of light, for example, the intensity is represented by the quantity of light energy falling normally per unit area in unit time. Similarly the intensity of sound at a point is given by the amount of wave energy striking per unit time per unit area about that point at right angles to the direction of flow. It will be seen later that this energy is proportional to

(*a*) the square of the amplitude of vibration,

(*b*) the density of the medium.

For the same source, same amplitude and the same medium the intensity of sound, like the intensity of light, falls off inversely as the square of the distance (Art. 22). It will also be seen later that the intensity is modified by the temperature and motion of the medium and the presence of other sonorous bodies in the neighbourhood.

Intensity regarded as a degree of sensation. When intensity is used to represent the degree of sensation perceived, it

becomes a physiological effect and falls outside the province of Physics. The word *loudness* is generally used in this sense, which though depending on the sensitiveness of the ear is found to increase and decrease with the intensity. Intensity is a physical quantity and can be stated numerically, whereas loudness, since it depends partly on the intensity and partly on the ear, is not susceptible of precise numerical statement.

Character, quality, timber or musical colour.

Character is that feature of musical sounds by means of which sounds of the same pitch and intensity emitted by different sources, a violin, a harmonium and a flute, for example, are distinguished from one another. We have seen that the physical cause of pitch is the frequency and that of intensity the amplitude of vibration of the source. The frequency and the amplitude being the same the only other aspect in which the sounding bodies can differ is the mode of their vibration at each instant, which is, therefore, the physical cause of character.

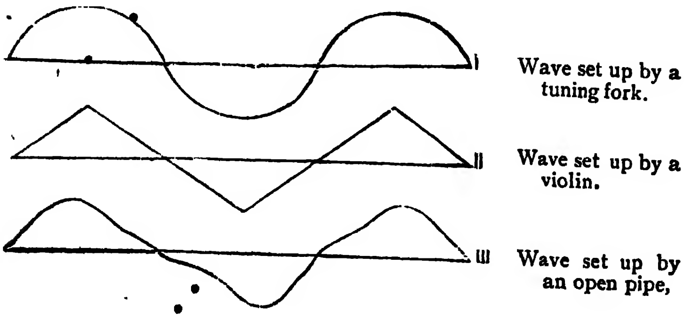


FIG. 9.

These waves are of the same amplitude and frequency but have different forms giving rise to the differances in the character of the sounds. This subject will be considered fully in a subsequent chapter.

CHAPTER II

ELASTICITY.

12. **Elasticity.** When a system of forces acting on a body does not produce any motion of the body as a whole, but simply causes a change in its shape or volume by producing relative displacements of its parts, the system of forces thus in equilibrium constitutes a *stress*, and the body is said to be in a state of *strain* When a stress acting on a body produces strain in it, a restoring force is in general brought into play by the relative displacements of its parts, tending to bring back the body to its unstrained condition. This property of a body in virtue of which the restoring force is brought into play is called its *elasticity*. Elasticity of a body, therefore, resists strain in it, and when strained it tends to restore the body to its initial state. A body is said to be perfectly elastic when (*a*) a given stress produces in it a definite strain, (*b*) the stress has got to be kept applied unchanged to maintain the strain, and (*c*) the body recovers its initial state precisely when the stress ceases to act.

13. **Limits of Elasticity**—Many substances are found to satisfy the conditions of perfect elasticity within certain limits and these limits are called the *limits of elasticity* of those substances.

14. **Hook's Law**—When an elastic body is distorted in shape or volume the stress required to distort it

which is equal and opposite to the restoring force brought into play, is proportional to the amount of distortion. This proportionality of the stress to the strain is known as *Hooke's law.*

$$Stress \propto strain,$$

$$= E \times strain, \text{ where } E \text{ is a constant.}$$

$$\therefore \quad \frac{Stress}{Strain} = \text{constant} = E$$

E is called the coefficient or modulus of elasticity of the substance.

15. **Volume elasticity.** Let a uniform hydrostatic pressure p, acting on a body of volume V per unit area of its surface, diminish the volume by v.

Then stress = pressure per unit area = p.

Volume strain = change in volume per unit volume = $\frac{v}{V}$

$$E = \textit{Coefficient of volume elasticity} = \frac{stress}{strain} = \frac{p}{v/V}$$

$$= \frac{pV}{v} \text{ units of force per unit surface.}$$

This applies to solids, liquids and gases. Liquids and gases, however, have no definite shape and, therefore, they possess only *volume elasticity*. Only solids possess *elasticity of shape*

16. **Young's modulus or modulus of longitudinal elasticity**. Let a metal wire of length L and cross section s, being stretched by a force F along its length, increase in length by l.

The stress = force per unit area of cross section = $\frac{F}{s}$

Strain = elongation per unit length $=\frac{l}{L}$

$\therefore$ $E=$ Young's modulus of the material of the substance $=\dfrac{\text{stress}}{\text{strain}}=\dfrac{F}{s}\div\dfrac{l}{L}=\dfrac{FL}{sl}$ units of force per unit surface.

In the case of a gas the value of the volume elasticity varies with the temperature conditions under which the changes take place.

(*a*) If the changes take place slowly under isothermal conditions, the heat generated during compression is given out, and heat is taken in during expansion, keeping the temperature constant. The relation between the pressure and volume of a gas at constant temperature is given by Boyle's law

$$PV=P_1V_1=P_2V_2=\text{constant.}$$

Let a very small increase in pressure from P to $P+p$ diminish the volume from V to $V-v$, where p and v are infinitely small quantities, so that their product is negligible.

Then $(P+p)(V-v)=PV.$

$$\therefore \quad PV-Pv+pV-pv=PV.$$

But pv being the product of two infinitely small quantities is negligible.

$$\therefore \quad Pv=pV \text{ or } P=\frac{p}{v/V}=\frac{\text{stress}}{\text{strain}}=E_i$$

where E_i is called the coefficient of isothermal elasticity of the gas.

(*b*) Again if the changes take place very quickly under adiabatic conditions, no heat is allowed to enter or leave the gas during expansion and compression. The

relation between the pressure and volume of a gas under such conditions is given by

$$PV^{\gamma}=P_1V_1^{\gamma}=P_2V_2^{\gamma}=\text{constant},$$

where $\gamma=\dfrac{\text{Sp. heat of the gas at constant pressure.}}{\text{Sp. heat of the gas at constant volume.}}$

Let the pressure be raised from P to $P+p$ and the volume changed from V to $V-v$.

Then $PV^{\gamma}=(P+p)(V-v)^{\gamma}=(P+p)V\gamma\left(1-\dfrac{v}{V}\right)^{\gamma}$

By applying Binomial theorem and neglecting higher powers of v, we get,

$$PV^{\gamma}=(P+p)\ V^{\gamma}\left(1-\gamma\frac{v}{V}\right)$$

$$=\left(PV^{\gamma}+pV^{\gamma}\right)\left(1-\gamma\frac{v}{V}\right)$$

$$=PV^{\gamma}-\gamma PvV^{\gamma-1}+pV^{\gamma}\text{, (neglecting } pv)$$

or $pV^{\gamma}=\gamma PvV^{\gamma-1}$

$\therefore\ pV=\gamma Pv$

$\gamma P=\dfrac{p}{v/V}=\dfrac{\text{stress}}{\text{strain}}=E_a=$ *coefficient of adiabatic elasticity of the substance.*

The result may also be obtained in another way

$$PV=C=\text{constant}.$$

$$\therefore\ \log P+\log V=\log C$$

$$\frac{dP}{P}+\frac{dV}{V}=0$$

$$P = -\frac{dP}{dV/V} = E_i$$

Since increase in pressure diminishes the volume, the changes are in opposite directions, and so if one is positive the other must be negative. dP and dV correspond to p and v considered previously

Again $PV^{\gamma} = C$

$\therefore \log P + \gamma \log V = \log C$

$$\frac{dP}{P} + \gamma \frac{dV}{V} = 0$$

$$\therefore \quad \gamma P = -\frac{dP}{dV/V} = E_a$$

These are extreme cases however. If there be partial transference of heat then elasticity will be intermediate between γP and P.

17. **Periodic and vibratory motions** The motion of a body is termed periodic when the series of movements through which it passes are repeated at regular intervals, and it is termed vibratory when in addition to being periodic it undergoes changes in direction continually. The motions of the hands of a watch and that of the earth relative to the sun are periodic. The motion of a simple pendulum and that of a vibrating plate are vibratory.

18 **Fundamental law of vibratory motion of elastic bodies.** When a steel plate fixed at one end is bent aside from its normal position A to A'', each displaced particle is acted on by a restoring force which urges the particle to its position of rest, and is proportional to its displacement. The plate thus springs back, but due to inertia of motion acquired, instead of coming to rest at A, goes

Fig. 10.

backward to A'. But there again it is similarly urged towards A and retraces its path, and thus goes on moving to and fro about its position of rest. The proportionality of the elastic or the restoring force to the displacement thus forms the fundamental law of vibratory motion of elastic bodies.

19. **Simple harmonic motion.** In studying vibratory motion, which is the cause of sound, we have to start with the simplest possible case of (a particle moving to and fro about its position of rest along a straight line, being acted on by a force, which constantly urges it towards the middle point of the path and is proportional to the displacement of the particle from that point. Such motion of a particle is called a *Simple harmonic motion*)—*S. H. M.*

20. **Transformation of mechanical energy into wave energy and its ultimate dissipation as heat.** The mechanical work done to bend the plate (fig 10.) is not lost but is stored up in the plate in the potential form, or the plate gains this amount of energy during the course of its displacement. Elastic or restoring forces are now brought into play which set the plate into motion towards A. The velocity of the plate is momentarily zero at A'' and whole of its energy is potential there, and as it moves back its velocity gradually increases and more and more of the potential energy becomes kinetic. The velocity is maximum at A and whole of the energy is kinetic there. The plate now passes A and moves to A' due the momentum acquired during its motion, and has to overcome the elastic resistance which opposes displacement. The velocity thus

decreases gradually and more and more of the kinetic energy becomes potential, and when the velocity is again zero at A'', whole of the energy again becomes potential. Thus if there be no loss of energy due to any cause, the *energy of vibration*, that is, the sum of the potential and kinetic energies of the plate will remain constant.

The vibrating plate does work in setting the layer of the surrounding medium into motion. The energy thus stored up in the layer is transferred from layer to layer onwards, and if there be no loss of energy in transmission, each layer will pass on its energy to the next layer undiminished. But the energy is dissipated as heat both in the plate and in the medium. During the transformation of energy from potential to kinetic and kinetic to potential the molecular friction of the plate produces heat which is dissipated away. The total energy of vibration thus diminishes gradually and the amplitude becomes smaller and smaller. It is also to be noted that the energy communicated by the source to the surrounding medium during each vibration is greater in denser media and the vibration of the plate will die out sooner in such cases. Again in the medium itself the friction between its molecules, however small, causes further dissipation of energy as heat. The whole of the energy is thus converted ultimately into heat.

21. Intensity falls off as $\frac{1}{r^2}$

In a homogeneous isotropic medium sound wave starting from a point source spreads out uniformly in the form of a sphere, of gradually increasing radius with the source as its centre. Neglecting the loss of energy due to

friction, and assuming that there is no accumulation of energy anywhere in the medium during transmission, the energy given out by the source per second is equal to the energy which crosses the surface of any concentric sphere per second.

Let E=energy given out by the source per second.

Then since the flow is normal everywhere across the surface of the sphere

I_1 =intensity at any point of a sphere of radius $r_1 = \dfrac{E}{4\pi r_1^2}$

I_2 =intensity at any point of a sphere of radius $r_2 = \dfrac{E}{4\pi r_2^2}$

$$\therefore \quad \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}.$$

The intensity varies as the square of the amplitude, and we also find that it falls off inversely as the square of the distance. Therefore the amplitude falls off inversely as the distance from the source.

$$I \propto a^2$$

$$\propto \frac{1}{r^2} \qquad \therefore a \propto \frac{1}{r}$$

CHAPTER III.

CIRCULAR MOTION AND SIMPLE HARMONIC MOTION.

22. Velocity and acceleration in uniform circular motion. Let a particle starting from P move round the circumference of a circle of radius r and centre O, with uniform speed v.

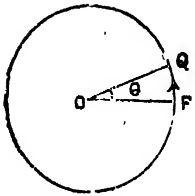


Fig. 12.

Let t=time taken by the particle to move from P to Q.

θ=the angle POQ,

ω=angular velocity of the particle.

Then $\theta=\omega t$.

$$PQ=vt=\theta r=\omega tr.$$

$$\therefore \quad \omega=\frac{v}{r}$$

A particle moving in a circle with a uniform speed is subjected to a constant force towards the centre, producing an acceleration v^2/r.

The velocity of the particle at any point of its path is along the tangent at that point, and therefore, the particle would move along the tangent. Hence in order to constrain the particle to move in the circle it must be acted on by a force, wich would constantly bend its path towards the centre, so that the particle is kept moving in the circle.

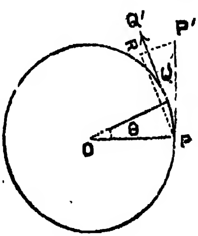


Fig 12

Let the particle P move to Q in time t.

The velocity at P is along PP' and let it be represented by the length PP'. The velocity at Q is along QQ' and let it be represented by the length QQ'.

The velocity of the particle in moving from P to Q has remained constant in magnitude but has changed in direction from PP' to QQ'. Draw PR parallel to QQ' and drop $P'R$ perpendicular to PR. Then to change PP' into PR we must add to it the vector $P'R$. In the limit when P and Q are infinitely near, $P'R$ becomes parallel to PO or at right angles to PP' or PR, and the chord PQ becomes equal to the are PQ.

Change in velocity per second = acceleration.

$$\therefore \quad \text{acceleration} = \frac{P'R}{t}.$$

Thus the force acting on P = mass $\times$ acceleration.

$$= \frac{mP'R}{t}, \text{ where } m = \text{mass}.$$

Again from similar triangles POQ and $PP'R$ we have

$$\frac{P'R}{PP'} = \frac{PQ}{OP}$$

$$\therefore \quad P'R = \frac{PP'.PQ}{OP}$$

But $PP' = v$, $PQ = vt$, $OP = r$

$$PR' = v^2 t$$

the accceleration towards the centre $= \dfrac{v^2}{r}$.

and the force towards the centre $= \dfrac{mv^2}{r}$

* Art. 79 General Physics.

23. Projection of a uniform circular motion gives a simple harmonic motion—*S. H. M.*

Take two diameters XOX' and YOY' at right angles to each other and drop perpendiculars PM and PN on them.

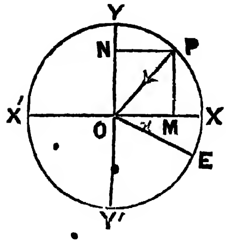


Fig. 13.

We have seen that P revolving round a circle with a uniform speed v, is acted on by a constant acceleration $\frac{v^2}{r}$ towards the centre. The component of this acceleration parallel to XOX'

$$=\frac{v^2}{r}\cos POM=\frac{v^2}{r}\frac{OM}{OP}=\frac{v^2}{r^2}OM.$$

Again the motion of P parallel to XOX' is identical with that of M. Since the velocity of M decreases as x increases, the acceleration is negative.

$$\therefore \text{ acceleration acting on } M \text{ along } MO=-\frac{v^2}{r^2}OM$$

$$=\mu OM$$

where μ is a constant and equal to $-\frac{v^2}{r^2}$

Thus the force acting on M=mass × acceleration

$$=-\frac{mv^2}{r^2}OM \text{ along } MO$$

$$=\mu_1 OM$$

where μ_1 is a new constant and equal to $-\frac{mv^2}{r^2}$

In other words M moves along XOX', being acted on by a force which urges it constantly towards O, the

middle point of the path, and which is proportional to the displacement of M form O. Therefore M executes a simple harmonic motion.

Similarly the component of motion P parallel to YOY' is identical with that of N, which is therefore acted on by an acceleration $= -\frac{v^2}{r}\cos PON = -\frac{v^2 ON}{r\,OP}$

$$= -\frac{v^2}{r^2} ON \text{ along } NO$$

$\therefore$ force acting on $N = -m\frac{v^2}{r^2}ON = \mu_1 ON$

Thus N also executes a $S.\ H.\ M.$

Similarly it may be shown that the projection of P on any diameter is a $S\ H.\ M.$, and the effect of the projection on a diameter is the same as that on any straight line parallel to it. The circle in which P moves is called the *auxiliary circle* or *the circle of reference.*

24. Vibration of a S. H. M. are isochronous. We thus see that if a force acting on a particle produces in it an acceleration μx, where μ is a constant and x the displaoement of the particle from a fixed point, then the particle executes a $S.\ H.\ M.$

Acceleration $= \mu\, x$.

But acceleration acting on the particle towards the fixed point

$$= -\frac{v^2}{r^2}x = -\frac{\omega^2 r^2}{r^2}x = -\left(\frac{2\pi}{T}\right)^2 x$$

$$\mu\, x = \left(\frac{2\pi}{T}\right)^2 x, \quad \text{or} \quad T = \frac{2\pi}{\sqrt{\mu}}.$$

π and μ are constants. Therefore T is constant and is independent of the amplitude, or the vibrations are isochronous, that is, are executed in equal times.

Musical instruments give out sounds of varying intensities depending on the force used in producing them, but though this force influences the intensity, it does not change the pitch, otherwise music would be practically impossible.

25. Equation of simple harmonic motion.

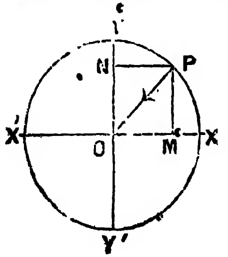


FIG. 14.

Let $x = OM$ = displacemment of M from the mean position O.

Then the equation

$x = r \cos \theta$...(i) defines the position of the particle M executing *S. H. M.* along XOX'.

Similarly $y = r \sin \theta$. (ii) defines the position of particle N executing *S. H. M.* along YOY'.

Equation (i) may also be represented as

$$x = r \cos \omega t$$

$$= r \cos \frac{2\pi}{T} t$$

$$= r \cos 2\pi n t \text{...(iii)}$$

$$\text{and } y = r \sin 2\pi n t \text{...(iv)}$$

where ω = tne angular velocity,

T = time-period,

n = frequency = $1/T$

Simple harmonic motion may, therfore, be defined as a motion in which the displacement from the mean point is proportional to the sine or cosine of an angle which varies as the time.

If θ be measured from some fixed diameter OE, making an angle e with OX, then the equations are changed as

$$x = r \cos (\theta - e).$$
$$y = r \sin (\theta - e).$$

Time-period or period is the time taken to execute one complete to and fro motion or the time taken by P to complete one rotation in the auxiliary circle.

Frequency is the number of complete to and fro motions executed per second $= \frac{1}{T}$, where n = the frequency, T = the time-period.

Amplitude is the maximum displacement of the particle from its mean position or the radius of the auxiliary circle.

Phase is the time or the fraction of the period which has elapsed since the last passage of the particle through its mean position in the positive direction or its angle equivalent θ swept by P since its last passage through OX.

Epoch, e which denotes the phase at the commencement of time or its time equivalent $\frac{eT}{2\pi}$, which defines the starting point of the particle, is called the epoch.

26. **Energy of a vibrating particle.** Let a particle of mass m execute a *S. H. M.* represented by $y = a \sin \omega t$. The velocity of the particle, is given by $\omega a \cos \omega t$. The kinetic energy of the particle, therefore, is $\frac{1}{2} m\omega^2 a^2 \cos^2 \omega t$, the maximum value of which is $\frac{1}{2} m\omega^2 a^2$. We have seen that the energy is usually partly kinetic and partly potential, the sum of the two being constant and when the kinetic energy is maximum the potential energy is zero. Therefore the total energy is given by

$$E = \frac{1}{2} m\omega^2 a^2.$$

27. The displacement curve of S. H. M.

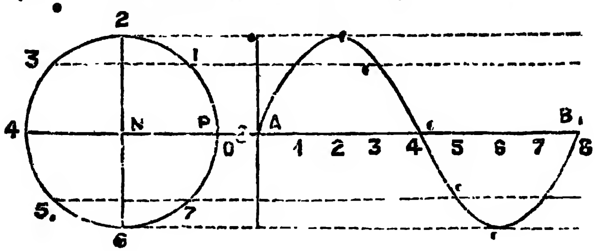


FIG. 15.

The successive stages of a simple harmonic motion are represented with the help of a displacement curve in which the abscissæ represent time and the ordinates the corresponding displacements. Let P revolve uniformly round a circle with O as its centre. Draw rectangular axes XOX' and YOY' through O. Then N, the projection of P on YOY' gives a simple harmonic motion. Divide the quadrants into the same number of equal parts, and draw straight lines through the points of division parallel to XOX', as shown in the figure. In the figure the circumference is divided into 8 equal parts. The time taken by P to move through each part is $T/8$, where T is the time-period. Take a length AB on the X-axis to represent time. Divide AB into eight equal parts. Then when P is at p on the circle, N is at the centre. Starting from this instant the displacement of N at intervals of $T/8$ are given by the ordinates at 1, 2, 3, 4 etc. between AB. The displacement curve, thus obtained, is identical with the well-known *sine-curve*, in which the abscissæ represent time and the ordinates the sine of an angle which itself varies with time.

28. Features of the curve.

1. Abscissæ represent time.

2. Ordinates represent displacements—upward ordinates displacements to the right and downward ordinates displacements to the left

3. Velocity is maximum where the slope of the curve, its inclination with the X-axis, is maximum, and it is zero at the horizontal part where the inclination or the slope is zero. The slope of the curve, therefore, represents the velocity.

4 The rate of change of velocity being the acceleration, the acceleration is zero where the curve is straight and maximum where the curvature is maximum.

29. The projections of a S. H. M on any straight line is another S H. M. Let P execute a simple harmonic motion along XOX'. Take another straight line AOB, making an angle θ with XOX' Drop PN perpendicular on AOB. Then the component of motion of P parallel to AB is identical with the motion of N along AOB.

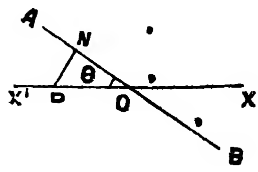


Fig. 16.

Force acting on P towards O

$\propto OP$

$= \mu \, OP$

where μ is a constant.

$\therefore$ component of this force parallel to $AB = \mu OP \cos\theta$

But $\cos\theta = \dfrac{ON}{OP}$

$\therefore$ the force acting on N towards $O = \mu\ ON$.

i e, the force acting on N is proportional to the displacement of N from O.

Thus N executes a $S\ H\ M$.

30. Two equal and opposite circular motions combine into a S H M.

Let P and Q revolve round a circle in opposite directions with the same angular velocity. Starting from any two points A and B, and moving opposite ways, they meet at a definite point A_1 and again at B_1, and come back again to A and B. A_1 and B_1 are two ends of a fixed diameter and P and Q pass each other at these points in each revolution. The circular motions of P and Q may be resolved along $A_1\ B_1$ and at right angles to it The components at right angles to $A_1\ B_1$ are equal and opposite, and those along $A_1 B_1$ are the same in magnitude and direction. The result of the combination, therefore, gives a $S.\ H.\ M.$ with amplitude equal to twice the radius of the the circle.

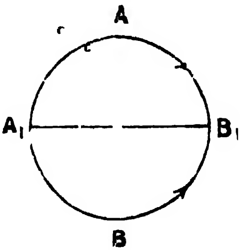


Fig 17

$X = a \cos \omega t$, $Y = a \sin \omega t$ is a circular motion represented by $X^2 + Y^2 = a^2$, so also, changing the sign of ω we have $X = a \cos \omega t$, $Y = -a \sin \omega t$ is a circular motion in the opposite direction. Adding the X—components, and also Y—components, we get for the resultant motion.

$$X = 2a \cos \omega t,\ Y = 0$$

that is, two opposite circular motions combine into a simple harmonic motion of double the amplitude.

CHAPTER IV.

COMPOSITION OF SIMPLE HARMONIC MOTIONS.

31. Two S. H. Ms, of the same period and executed along the same line,

Geometrical method. Let P and Q revolve in the same direction round concentric circles of radii equal to OP and OQ with the same angular velocity. Draw rectangular axes XOX', and YOY' through O. Then N and M, the projections of P and Q on XOX', give two *S. H. Ms* of the same period with amplitudes equal to OP and OQ, and phase difference equal to the angle POQ.

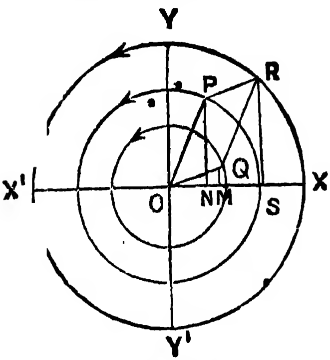


Fig. 18

Complete the parallelogram $OPRQ$ with OP and OQ as adjacent sides. Join OR.

Then since PR is parallel to OQ

and RS is parallel to PN

$$ON = MS$$

$$\therefore \quad OM + ON = OM + MS = OS.$$

The displacements of M and N being along the same straight line, the resultant displacement $= OM + ON = OS$.

Since P and Q move with the same angular velocity, the angle POQ remains constant, and therefore OR, the diagonal of the parallelogram, also revolves round O with the same angular velocity. Thus the projection of R on XOX' gives S, executing *S. H. M.* of the same period and having for its displacement OS, which is the resultant of the displacements of the component *S. H. Ms.* The result of the composition, therefore, is a third *S. H. M.* of amplitude equal to OR

Again $OR^2 = OP^2 + OQ^2 + 2\,OP.OQ \cos POQ$

and phase = the angle XOR.

(*a*) when the phase difference is zero,

angle $POQ = 0$, and therefore $\cos POQ = 1$

$\therefore \quad OR = OP + OQ$

(*b*) when the phase difference is 180°

angle $POQ = \pi$, and therefore $\cos POQ = -1$

$\therefore \quad OR = OP - OQ.$

If in addition $OP = OQ$, there is no motion.

Analytical method. Let the two *S. H. Ms* be represented by

$$y_1 = a_1 \sin(\theta - e_1)$$

$$y_2 = a_2 \sin(\theta - e_2)$$

The result of their composition is given by

$$\begin{aligned} y = y_1 + y_2 &= a_1 \sin(\theta - e_1) + a_2 \sin(\theta - e_2) \\ &= a_1 (\sin\theta \cos e_1 - \cos\theta \sin e_1) \\ &\qquad + a_2 (\sin\theta \cos e_2 - \cos\theta \sin e_2) \\ &= \sin\theta\,(a_1 \cos e_1 + a_2 \cos e_2) \\ &\qquad - \cos\theta\,(a_1 \sin e_1 + a_2 \sin e_2) \end{aligned}$$

Put $R \sin E = a_1 \sin e_1 + a_2 \sin e_2$,

$R \cos E = a_1 \cos e_1 + a_2 \cos e_2$,

Then $y = R\,(\sin\theta \cos E - \cos\theta \sin E)$

$= R \sin(\theta - E)$

This is the equation of a third *S. H. M.* of amplitude R and epoch E.

R is given by the relation,

$R^2 = R^2(\cos^2 E + \sin^2 E) = R^2 \cos^2 E + R^2 \sin^2 E$

or $R^2 = (a_1 \cos e_1 + a_2 \cos e_2)^2 + (a_1 \sin e_1 + a_2 \sin e_2)^2$

$= a_1^2 + a_2^2 + 2a_1 a_2 \cos(e_1 - e_2)$;

and E is given by the relation,

$$\tan E = \frac{a_1 \sin e_1 + a_2 \sin e_2}{a_1 \cos e_1 + a_2 \cos e_2}.$$

The S. H. Ms may also be represented as

$y_1 = a_1 \cos(\theta - e_1)$,

$y_2 = a_2 \cos(\theta - e_2)$,

$\therefore\ y = y_1 + y_2$

$= a_1 \cos(\theta - e_1) + a_2 \cos(\theta - e_2)$

$= a_1\,(\cos\theta \cos e_1 + \sin\theta \sin e_1) + a_2\,(\cos\theta \cos e_2 + \sin\theta \sin e_2)$

$= \cos\theta\,(a_1 \cos e + a_2 \cos e_2) + \sin\theta\,(a_1 \sin e_1 + a_2 \sin e_2)$

Put $R \cos E = a_1 \cos e_1 + a_2 \cos e_2$,

and $R \sin E = a_1 \sin e_1 + a_2 \sin e_2$,

Then $y = R\,(\cos\theta \cos E + \sin\theta \sin E)$

$= R \cos(\theta - E)$

R and E are obtained as in the previous case.

32. Composition of two rectangular S. H. Ms of the same period.

Geometrical method. Let M and N execute *S. H. Ms.* along two rectangular axes XOX' and YOY', O being the central positions of both. Draw straight lines through X, O, X', Y and Y' parallel to XOX' and YOY', and describe the auxiliary circles on diameters parallel to XOX' and YOY' as shown in the figure. The projection of Q, describing the lower circle on XOX', gives M executing *S. H. M.* along XOX', and the projection of P, describing the right-hand circle on YOY', gives N executing *S. H. M.* along YOY'.

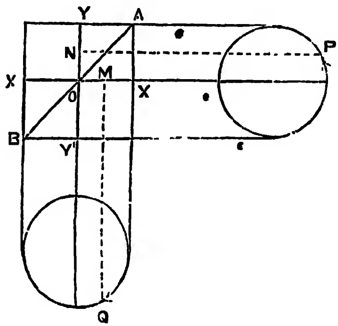


Fig. 19

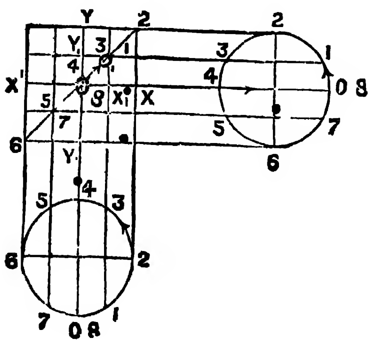


Fig. 20.

Divide the quadrants of the circles into the same number of equal parts and draw straight lines through

the points of division on the lower circle parallel to YOY' and similarly through the points of division on the right-hand circle parallel to XOX'.

Case 1. Phase difference $=0$. In this case when M is at O and moves along the positive direction OX, N is also at O and moves along the positive direction OY. The corresponding positions of P and Q are given by the numeral 0 on the circles. Place numerals 1, 2, 3 etc. on the circles as shown in figure 20. When P and Q move to the positions given by 1 on the circles, M moves to X_1 and N to Y_1 and the resultant motion is given by oa_1. When P and Q will complete one revolution, the resultant motion of M and N will give the diagonal AB traced twice in opposite directions.

Phase difference $=\pi$. In this case when M is at O and moves along the positive direction OX, N is also at O but moves is the negative direction OY', it having already executed half of its motion. The numerals on one circle, therefore, are to be shifted through π, and those on the other circle left where they are. Proceeding as before we find that the resultant in this case is the diagonal CD, traced twice in opposite directions.

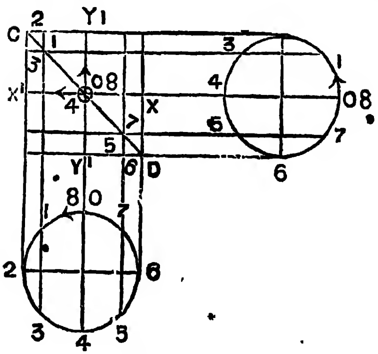


Fig. 21

Phase difference $=\pi/4$. In this case when N is at O and moves along the positive direction OY, M has already executed $\frac{1}{8}$th of its motion, and moves towards X. The numerals on one circle, therefore, are shifted through $\pi/4$ and those on other circle left where they are. Proceeding as before we get for the resultant an oblique ellipse shown in the figure.

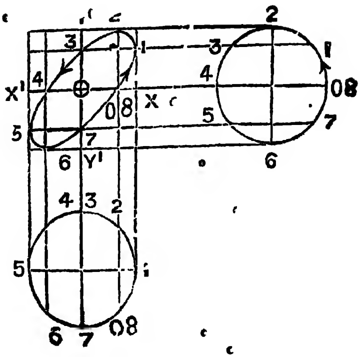


Fig. 22.

Phase difference $=\pi/2$. In this case we get an ellipse with XOX' and YOY' as its axes.

If in addition $OP=OY$, the ellipse reduces to a circle.

Analytical method.

Let the two *S. H. Ms.* be represented by

$x=a\sin(\theta+\alpha)$......(i)

$y=b\sin\theta$..............(ii)

where a and b are the amplitudes and α is the phase difference.

The equation of the resultant motion is obtained by eliminating θ between these two equations.

From (ii) we have $\frac{y}{b}=\sin\theta$

From (i) $\frac{x}{a}=\sin(\theta+\alpha)=\sin\theta\cos\alpha+\cos\theta\sin\alpha$

$$=\frac{y}{b}\cos\alpha+\sqrt{1-\frac{y^2}{b^2}}\sin\alpha$$

$$\therefore \left(\frac{x}{a}-\frac{y}{b}\cos\alpha\right)^2=\left(1-\frac{y^2}{b^2}\right)\sin^2\alpha$$

or $\frac{x^2}{a^2}+\frac{y^2}{b^2}(\cos^2\alpha+\sin^2\alpha)-\frac{2xy}{ab}\cos\alpha-\sin^2\alpha=0.$

Multiplying by a^2b^2 we get

$b^2x^2+a^2y^2-2ab\,xy\cos\alpha-a^2b^2\sin^2\alpha=0$ (iii)

This is the equation of an ellipse inclined to the co-ordinate axes.

(1) *When* $\alpha=0$

Equation (iii) reduces to $(bx-ay)^2$, which represents a pair of coincident straight lines through O, and lying in the first and the third quadrants. The physical significance of a pair of coincident straight lines is that the line AB, in fig. 19, is traced twice in opposite directions in the time-period.

(2) *When* $\alpha=\pi$

Equation (iii) reduces to $(bx+ay)^2$ which again represents a pair of coincident straight lines passing through O and lying in the second and the fourth quadrants. It represents CD in fig. 21.

(3) *When* $\alpha=\frac{\pi}{2}$

equation (iii) reduces to

$b^2x^2+a^2y^2-a^2b^2=0$, which represents the regular ellipse with semi-axes a and b along the co-ordinate axes. If in addition $b=a$ the equation reduces to a circle.

(4) *When* $\alpha=\frac{\pi}{4}$

equation (iii) becomes

$$b^2x^2 - 2ab\,xy/\sqrt{2} + a^2y^2 - a^2b^2/2 = 0$$

which represents the oblique ellipse in fig. 22

Periods 1 : 2.

Graphiacl method :—*Phase difference*=0.

Let the period of the *S. H. M.* along *XOX'* be twice the period of the *S. H. M.* along *YOY'*.

Divide the quadrants of the lower circle into n equal parts, and those of the right hand circle into $2n$ equal parts, and proceed as before. The result of the composition is the curve in fig. 23.

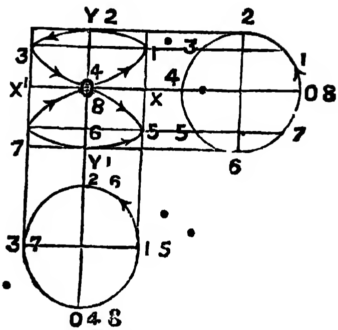


Fig. 23.

Phase difference$=\frac{\pi}{2}$. Shift the numerals on the lower circle through 90° and leave those on the other unaltered. The result of the composition in this case is a parabola which is traced twice in the time-period.

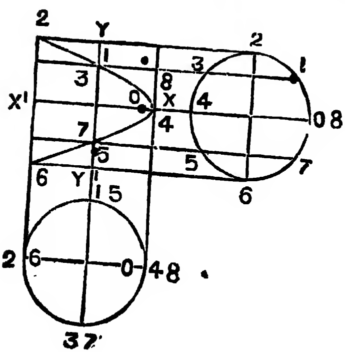


Fig. 24.

For any other phase difference shift the numerals on the lower circle through the corresponding angle, those on the other circle remaining in the standard position.

Analytical method. Let the *S. H. Ms* be represented by equations

$x = a \sin(2\theta + \alpha)$......(i)

$y = b \sin\theta$.............(ii)

From (ii) $\frac{y}{b} = \sin\theta.$

From (i) $\frac{x}{a} = \sin(2\theta + \alpha),$

$$= \sin 2\theta \cos\alpha + \cos 2\theta \sin\alpha$$

$$= 2\sin\theta\cos\theta\cos\alpha + (1 - 2\sin^2\theta)\sin\alpha$$

$$= 2\frac{y}{b}\sqrt{1 - \frac{y^2}{b^2}}\cos\alpha + \left(1 - \frac{2y^2}{b^2}\right)\sin\alpha$$

or $\left\{\frac{x}{a} - \left(1 - \frac{2y^2}{b^2}\right)\sin\alpha\right\}^2 = \frac{4y^2}{b^2}\left(1 - \frac{y^2}{b^2}\right)\cos^2\alpha$

or $\left(\frac{x^2}{a^2} - \frac{2x}{a}\sin\alpha + \sin^2\alpha\right) + \frac{4y^2}{b^2}\left(\frac{y^2}{b^2} + \frac{x}{a}\sin\alpha - 1\right) = 0$

$\therefore \frac{4y^2}{b^2}\left(\frac{y^2}{b^2} + \frac{x}{a}\sin\alpha - 1\right) + \left(\frac{x}{a} - \sin\alpha\right)^2 = 0$......(iii)

1. *When* $\alpha = 0$ (iii) reduces to

$\frac{4y^2}{b^2}\left(\frac{y^2}{b^2} - 1\right) + \frac{x^2}{a^2} = 0$ which is the equation of the curve in fig. 23.

2. *When* $\alpha = \frac{\pi}{2}$ (iii) reduces to

$$\left(\frac{2y^2}{b^2} + \frac{x}{a} - 1\right)^2.$$

which represents two coincident parabolas, represented by the curve in fig. 24. the equation of which is given by

$$y^2 = -\frac{b^2}{2a}(x - a).$$

The physical significance of a pair of coincident parabolas is that the same parabola is traced twice in the time-period.

33. Periods of other ratios. For any values of the periods, either commensurate or incommensurate, the graphical method gives the result easily. For example if the periods be in the ratio of 1 : 3, divide the quadrants of one circle into n equal parts and those of the other into $3n$. Again if the ratio be 2 : 3, divide the quadrants of one circle into $2n$ equal parts and those of the other into $3n$ and proceed as before. The analytical method however soon becomes very complicated, the curves being complex and of higher degrees than second. When the ratio, for example, is 1 : 3, we get the result by eliminating θ between the equations,

$$x = a \sin 3\theta$$
$$y = b \sin \theta$$

which gives an equation of the third degree.

34. Lissajous' figures :—The vibrating prong of a tuning fork executes *S. H. M.* and the combination of two *S. H. Ms.* previously considered can be experimentally illustrated by means of two forks.

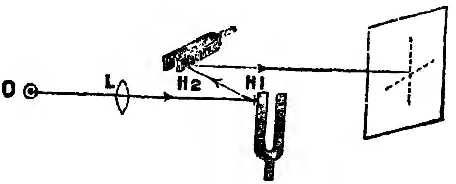


Fig. 25.

A beam of light from a powerful small source O after passing through a lens L, gets reflected from a small

mirror H_1, attached to the end of the prong of a tuning fork to a second mirror H_2, attached to the end of the prong of another fork, and thence to the screen. The forks are so clamped that one of them vibrates in the horizontal plane and the other in the vertical, and the mirrors are attached parallel to the flat of the prongs. By carefully adjusting L the beam of light is brought to a focus on the screen. ($OL=u$, $LH_1+H_1H_2+H_2S=v$; u, v being conjugate focal distances and $H_2S=$ distance of H_2 from the screen). If one of the forks remains steady and the other vibrates, the spot of light on the screen moves to and fro along a straight line presenting the appearance of a luminous line due to persistence of vision. If both the forks virbate, the spot of light exhibits luminous curves of different forms corresponding to the combination of two *S. H Ms* at right angles to each other of different amplitudes and phases. These curves may also be seen by substituting a telescope or naked eye for the screen. The curves became first known by Lissajous' experiment and are called *Lissajous figures.*

35. **Helmholtz's vibration microscope.** Another method of seeing these curves is by means of the vibration microscope arrangement devised by Helmholtz. The object glass of a compound microscope, of about 2 to 3 inches focus, is attached to the end of the prong of a tuning fork clamped horizontally, and the eye-piece is clamped steadily above it, so that a scratch on the end of the prong of another fork clamped vertically, is seen in

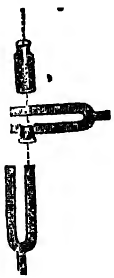


Fig 26.

the field of view of the eye-piece. The forks are so clamped that while the lower fork vibrates in the horizontal plane the upper one vibrates in the vertical. By suitably adjusting the amplitudes and the phases of the *S. H. Ms.* executed by the forks the corresponding curves resulting from their combination are seen, in the field of view of the eye-piece.

CHAPTER V.

WAVE MOTION.

36. **Wave motion.** When we have got a number of bodies linked together by elastic chains, we cannot displace any one of them without disturbing the others. The motion imparted to one is communicated through the chains to others, and they in their turn are set into motion. We may regard an elastic medium to consist of very fine particles with elastic connections between them. If we impart a periodic motion to any paticle, its energy of motion, will be communicated to the adjecent particles surrounding it, which will thus execute similar periodic motions and in doing so will in their turn transfer their energy to the neighbouring particles sorrouning them and so on; the motion imparted to a particle will move uniformly in all directions from particle to particle with a definite velocity. The propagation of a disturbance of this kind due to transfer of energy from particle to particle is called a *wave motion*

37. **Longitudinal progressive waves.** Progressive waves in which the particles of the medium through which they advance, execute *S. H. Ms.* about their positions of rest along the line of advance of the waves, are called *longitudinal waves.*

Sound waves are longitudinal. We have seen that when a body vibrates, its energy of motion is transferred to the adjacent air particles, which thus begin to vibrate

similarly, and in doing so communicate their energy to the particles in their front, which in their turn take up the motion and so on. The displacement set up at the source thus moves on uniformly in all directions from particle to particle of the medium with a definite velocity. Each particle executes a *S. H. M.* about its position of rest along the direction of propagation of the disturbance, giving rise to a train of waves consisting of alternate compressions and rarefactions.

38. **Transverse progressive waves**—Progressive waves in which the particles of the medium through which they advance execute *S. H. Ms.* about their positions of rest at right angles to the line of advance of the waves; are called *transverse progressive waves.*

Water waves and rope waves are transverse—In the case of water waves we have seen that the particles of water through which the waves advance, practically move up and down, while the waves are propagated horizontally along the surface. The particles of water execute *S. H. Ms.* at right angles to the line of advance of the disturbance, giving rise to a train of waves consisting of alternate crests and troughs. See art. 8.

39. **Waves travelling along a stretched flexible string.** To follow clearly how transverse waves are set up and propagated, let us take a stretched flexible string fixed at *P* and free at the other end *Q* at rest. Let *Q* execute a *S. H. M.* When *Q* moves towards *X*, if the string had been rigid, it would have moved as a whole as shown by the dotted straight line *XP*. But the string is

elastic and hence when Q moves, its energy of motion is communicated to the next particle, which therefore moves

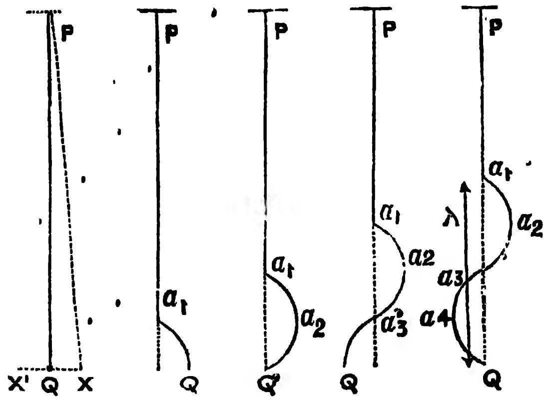


Fig. 27.

after Q. This particle similarly sets the third particle into motion after it and so on The motion imparted to Q is thus transferred from particle to particle and moves along the string with a definite velocity. By the time Q goes to X, the disturbance has moved upto a_1, and the particle at a_1 is just on the point of moving towards the right and reacting in its turn on the next particle. The disturbance if left to itself will continue moving up along the string. Q is momentarily at rest at X and all its energy is potential there. Then as it begins to move back it gains in kinetic energy; the particles between a_1 Q are still moving towards the right. Each particle will in its turn come to rest momentarily at its position of maximum displacement and then begin to move backwards. When Q has come

back to its inital position, the disturbance has moved upto Qa_2a_1 the particles between a_1 a_2 moving to the right and those between a_2Q to the left. Next as Q moves to X' and then back to its inital position the disturbance advances as shown by the curves $Qa_3a_2a_1$ and Qa_4a_3 a_2a_1 in fig. 27, The disturbance advances through a distance λ during one complete vibration of Q. Again it will be seen that Q and a_1 are both passing through their mean position of rest to the right. They are therefore in the same phase; and a_3 is passing through its mean position to the left and therefore a_3 and Q are in opposite phase. The disturbance $Q\ a_4a_3a_2a_1$ set up in the time-period constitutes a complete wave and the length λ of the wave is called the *wave-length*.

As Q repeats its *S. H. M.* the wave $Qa_4a_3a_2a_1$ advances up along the string being followed by similar waves.

Let V = volocity of propagation of the waves;

λ = wave length;

T = time period;

n = frequency of vibration;

Then $\lambda = VT$, $V = n.\lambda$.

40. Plane and spherical waves. Wave-front. Ray.

Let us now imagine an infinite number of similar and and parallel elastic strings having their ends attached to to the same plane at right angles to their lengths. A periodic motion imparted to the plane, for example, a to and fro motion parallel to itself, will cause a transference of the motion from particle to particle in each string. At

a certain instant the disturbance will have travelled to the same distance along each string, and therefore, *the locus of those points which are on the point of being disturbed* or the *wave-front* will be a *plane* parallel to the plane to which the end of the strings are attached.

Again let us imagine an infinite number of similar elastic strings to start out from the same point and diverge uniformly in all directions. Any periodic motion imparted to this centre will be transferred from particle to particle in each string. At a certain instant the disturbance will have moved to the same distance along each string, and therefore, the *locus of these points which are on the point of being disturbed* or the *wave-front* will be a *sphere* with the point of disturbance as its centre.

In both cases we may imagine elastic strings to fill up all the space surrounding the centre of disturbance, and we get the case of a continuous medium with a series of plane or spherical waves propagating through it.

Ray. The line of advance of the waves which is always at right angles to the waves is called a *ray*.

41. Graphical representation of wave motion.

Transverse progressive waves.—In the displacement curves of *S. H. M.* we considered the characteristics of the *S. H. M.* of a single particle. We have now to consider the arrangements of all the particles of the medium executing *S. H. M.s* at a given instant as the waves pass them. Let the undisturbed particles of the medium be represented by a row of equidistant points along the *X*-axis. We have seen that the line passing through the

displaced particles at any instant is a sine-curve and is, therefore, similar to the displacement curve of *S. H. M* already considered.

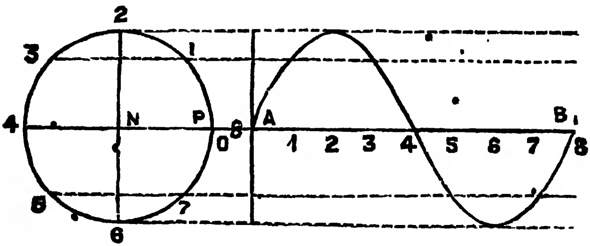


Fig 28.

Characteristic features.—All the particles through which the waves advance execute *S. H. Ms* about their mean positions at right angles to the line of advance of the waves, the period and amplitude being the same but the phase changing continuously from particle to particle with distance along the direction of wave propagation. The arrangement of the particles at any instant is in the form of alternate similar crests and troughs. The troughs are reversed copies of the crests, and hence the arrangement repeats itself at intervals of λ, and the physical conditions of any particle,—its velocity, acceleration, amplitude, phase and state of density—at any instant are identical with those of other particles at intervals of λ, at the same instant. The waves consisting of alternate crests and troughs move onwards with a definite velocity.

Combined velocity and displacement curves.

As explained on page 10, the combined velocity and

displacement curves of the partices of a medium disturbed by a progressive wave motion are given in fig. 29.

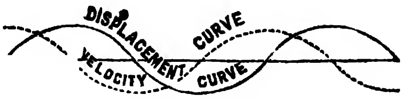


Fig. 29

Longitudinal progressive waves and their essential features.

All the particles through which the waves advance execute *S. H. Ms.* about their mean positions along the line of advance of the waves, the period and amplitude being the same, but the phase changing continuously from particle to particle with distance along the direction of wave propagation. The arrangement of the particles at any instant is in the form of alternate compressions and rarefactions, the velocites of the particles being along the direction of wave propagation during compression and in the opposite direction during rarefaction. The displacements of the particles at any instant are represented by a sine curve, on the convention that upward ordinates represent displacements to the right and downward ordinates displacements to the left. The compressions are all similar and so also the rarefactions, and thus this arrangement of particles repeats itself at intervals of λ, and the physical conditions of any particle,—its velocity, acceleration, amplitude, phase and state of density—at any instant are indentical with those at intervals of λ, at the same instant. The waves consisting of alternate compressions and rarefactions move onwards with a definite velocity.

42. General equation of wave motion.

Each particle executes a *S. H. M.* and its motion is transferred from particle to particle with a definite velocity.

The general equation of a *S. H. M.* is given by

$y = a \sin(\theta - e)$...(1) where y = displacement of the particle;
or $y = a \cos(\theta - e)$

a = amplitude;
θ = phase;
and e = epoch.

e, the epoch of a particle depends on x, its distance from the origin measured along X-axis, the line of advance of the waves. The phase changes continuously along X at such a rate that when 'x' increases through λ the phase changes through 2π.

Therefore $\frac{e}{2\pi} = \frac{x}{\lambda}$ or $e = \frac{2\pi x}{\lambda}$.

also $\theta = \omega t = \frac{2\pi t}{T} = \frac{2\pi Vt}{\lambda}$;

where T is the time period and V the velocity.
Substituting these values for e and θ in (1)
we get

$$y = a \sin\frac{2\pi}{\lambda}\left(Vt - x\right) \quad \dots (2)$$

which is, therefore, the general equation of wave motion.

The general equation may also be represented as

$$y = a \cos\frac{2\pi}{\lambda}\left(Vt - x\right).$$

where y = displacement of a particle at some instant,

a = amplitude,

V = velocity of transmission of the wave,

x = distance of the point from the origin,

t = time which has elapsed since the particle last passed through its mean position in the positive direction.

Velocity of the particle at this instant $= \frac{dy}{dt}$,

and its acceleration $= \frac{d^2y}{dt^2}$.

43. Relation between the particle velocity and acceleration, pressure, stress, strain and elasticity in longitudinal wave motion.

Let us consider the motion of a very thin layer of the medium at right angles to the line of advance of the waves.

Fig 30

Let XX' be the line of advance of the waves and AB a very thin layer of the medium at right angles to it and of unit cross section,

Let x = distance of A from the origin at any given instant.

$x + dx$ = distance of B from the origin at the same instant.

$\therefore$ volume of the layer $= dx$

Again let y = displacement of A after time dt,

$y + dy$ = displacement of B after time dt.

Then the distance between the faces A and B now becomes $(x+dx+y+dy)-(x+y)=dx+dy$

$\therefore$ volume of the layer now $=(dx+dy)$

Hence the change in volume undergone by volume dx in time $dt=(dx+dy)-dx=dy$

$$\therefore \quad \textit{volume strain} = \frac{dy}{dx} \quad . \quad . \qquad (1)$$

Velocity and acceleration.

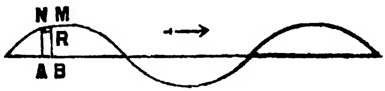


Fig. 31.

Let XX' be the line of advance of the waves. Consider two very near particles at A and B on XX'.

The displacement of the particle at A changes from the value AN to BM in the time during which the waves advance through the distance AB.

Let V = velocity of propagation of the waves ;

t = time taken by the waves to move through AB ;

v = average velocity of the particle during this time t.

Then $Vt=AB$;

$vt=(BM-AN)=RM$

or $\dfrac{vt}{Vt}=\dfrac{v}{V}=\dfrac{RM}{AB}=\dfrac{dy}{dx}$

When AB is infinitely small the average velocity becomes equal to the velocity of the particle at N.

Velocity of the particle at $N=v=V\dfrac{RM}{AB}$

But $\frac{RM}{AB} = \tan \alpha =$ slope ; where $\alpha =$ inclination of the curve to the X-axis at N.

$\therefore v = V \tan \alpha$ . . . (i)

acceleration at $N = \frac{V(\tan \alpha - \tan \alpha')}{\text{time}}$

$$= \frac{V(\tan \alpha - \tan \alpha')}{\frac{AB}{V}}$$

$$= V^2 \times \text{curvature}...(ii)$$

Again as shown in the previous article

$\frac{dy}{dx} =$ volume strain.

The stress which produces this strain is given by the excess of pressure at A over the normal pressure and let this be equal to p.

Stress $= E \times$ strain

or $p = E \times$ strain

strain $= \frac{p}{E}$;

Therefore $\frac{v}{V} = \frac{p}{E}$. But $\frac{v}{V} = \frac{dy}{dx}$. $p = E\frac{dy}{dx}$.

CHAPTER VI.

PROGRESSIVE WAVES.

44. Velocity of longitudinal waves in an elastic medium.

Let XX' be the line of advance of the waves and AB a very thin layer of the medium taken at right angles to XX'.

Fig 32

The particles of the medium through which the waves advance execute *S. H. Ms.* along straight lines parallel to XX'. The layer AB of the medium being very thin it moves backwards and forwards along XX', and the force per unit area which causes this motion at any instant is the difference of pressure at the two faces A and B of the layer. Again we have from Dynamics

force = mass × acceleration . . (

Let at any given instant

$P+p_1$ = pressure per unit area at A;

$P+p_2$ = pressure per unit area at B;

v_1 = particle velocity at A,

v_2 = particle velocity at B;

ρ = density of the medium;

V = velocity of transmission of the waves.

Then force per unit area $= p_1 - p_2$.

Mass of the layer of unit cross section = volume × density $= AB\rho$.

Its acceleration $= \dfrac{v_1 - v_2}{\dfrac{AB}{V}}$

The velocity changes from v_1 to v_2 in the time during which the waves advance through the distance AB.

Therefore from (1) we have

$$p_1 - p_2 = \frac{\rho . AB . V(v_1 - v_2)}{AB}$$

$$\text{or } \frac{p_1 - p_2}{v_1 - v_2} = \rho V$$

Again we have the relation

$$\frac{p}{v} = \frac{E}{V} = \text{constant [Art. 43.]}$$

$$\therefore \quad \frac{p_1}{v_1} = \frac{p_2}{v_2} = \frac{E}{V} = \frac{p_1 - p_2}{v_1 - v_2}$$

Hence we have $\dfrac{E}{V} = \rho V$ or $V^2 = \dfrac{E}{\rho}$

That is, $V = \sqrt{\dfrac{E}{\rho}}$ or the velocity depends only on the elasticity and density of the medium.

Alternative proof :—As before we have

$p_1 - p_2 =$ mass × acceleration. Let $AB = dx$

But $p_1 - p_2 = \dfrac{dp}{dx} dx = E \dfrac{d^2y}{dx^2} dx$; $\left[\because p = E \dfrac{dy}{dx} \right]$

mass $= \rho dx$;

and acceleration $=\frac{d^2y}{dt^2}$, where $y=a\sin\frac{2\pi}{\lambda}(Vt-x)$ in the general equation of wave motion.

Thus we have

$$E\frac{d^2y}{dx^2}dx=\rho\frac{d^2y}{dt^2}dx \quad . \quad (1)$$

But $\frac{dy}{dt}=a\frac{2\pi}{\lambda}V\cos\frac{2\pi}{\lambda}(Vt-x)$;

$\frac{d^2y}{dt^2}=-a\frac{4\pi^2}{\lambda^2}V^2\sin\frac{2\pi}{\lambda}(Vt-x)$;

and $\frac{dy}{dx}=-a\frac{2\pi}{\lambda}\cos\frac{2\pi}{\lambda}(Vt-x)$;

$\frac{d^2y}{dx^2}=-a\frac{4\pi^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(Vt-x)$.

Substituting these values of $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{dt^2}$ in (1) we have

$$E=\rho V^2$$

That is $V^2=\frac{E}{\rho}$

or $V=\sqrt{\frac{E}{\rho}}$

45 **Velocity of sound in air.** Sound waves are longitudinal, and therefore, the velocity of their propagation in air is given by

$V=\sqrt{\frac{E}{\rho}}$; where $E=$coefficient of volume elasticity of air, and $\rho=$its density.

If the compressions and rarefactions take place, slowly under isothermal conditions so that the temperature remains constant, then $E=P$, the pressure of air.

Again if the changes take place under adiabatic conditions, that is, too quickly to allow of any transference of heat from particle to particle then

$$E=\gamma P\ ;\ \text{where}\ \gamma=\frac{Cp}{Cv}.\quad \text{Art. 16.}$$

On the other hand if there be partial transference of heat by conduction and radiation during the changes then

E lies between P and γP.

V can not be greater than $\sqrt{\frac{\gamma P}{\rho}}$

V can not be less than $\sqrt{\frac{P}{\rho}}$

46. Newton's value and Laplace's correction. The velocity of sound in air was theoretically investigated first by Newton, who applied Boyle's law to deduce his result, and therefore assumed the changes to be isothermal. Newton's expression for velocity is $V=\sqrt{\frac{P}{\rho}}$. But the value obtained from this expression was found by Newton to be less by about 1/8th of the actual value, and he tried to explain the discrepancy by assuming that the molecules of the medium occupied about 1/8th the linear distance traversed, and that sound moved instantly through the molecules and took time to pass through the interspaces only.

The real cause of the discrepancy was given by Laplace, who assumed that the changes took place under adiabatic conditions. His expression for the velocity is given by

$$\sqrt{\frac{\gamma P}{\rho}}$$

and the value obtained from this expression agrees with experimental results.

Laplace's assumption is corroborated by Stoke's argument to the effect that if the rise in temperature during compression and its fall during expansion were to any sensible exten smoothed down by partial radiation and conduction, the the available amount of energy for wave transmission woul diminish and would lead to the intensity falling off much more rapidly than inversely as the square of the distance.

Velocity is independent of λ and is therefore the same for waves of all frequencies.

Newton's value expressed as $\sqrt{gH}$

In the expression $V=\sqrt{\frac{P}{\rho}}$

$P=g\rho H$, where H=height of the homogeneous atmosphere, and g=acceleration due to gravity.

Thus $V=\sqrt{\frac{g\rho H}{\rho}}=\sqrt{gH}=\sqrt{2g.\frac{H}{2}}$=velocity acquired by a body falling in vacuo through half the height of the homogeneous atmosphere.

47. Effects of pressure, temperature and moisture on V.

Change in pressure. The temperature remaining constant any change in pressure produces corresponding change in volume. Let P and ρ be the initial pressure and density, P_1 aud ρ_1 be the new pressure and density, and v and v_1 the volumes corresponding to P and P_1 respectively.

We have $V_1 = \sqrt{\frac{\gamma P_1}{\rho_1}}$

But $Pv = P_1 v_1$ $\quad P_1 = \frac{Pv}{}$

nd $= \frac{\rho}{\rho_1} = \frac{v_1}{v}$ $\therefore$ $\frac{}{\rho_1} = \frac{v_1}{\rho v}$

$$\therefore \quad V_1 = \sqrt{\frac{\gamma P_1}{\rho_1}} = \sqrt{\frac{\gamma Pv.v_1}{v_1 v\rho}} = \sqrt{\frac{\gamma P}{\rho}} = V$$

That is V is *independent of pressure.*

Change in temperature. The pressure remaining onstant any increase in temperature produces correspond-ig expansion and change in density.

Let velocity at $t^\circ C = Vt = \sqrt{\frac{\gamma P}{\rho t}}$

and velocity at $0^\circ C = V_0 \sqrt{\frac{\gamma P}{\rho_0}}$

where ρ_t and ρ_0 are the densities of air at $t^\circ C$ and $0^\circ C$ espectively.

But $\frac{\rho_t}{\rho_0} = \frac{v_0}{v_t}$ or $\rho_t = \frac{\rho_0 v_0}{v_0 (1 + \alpha t)}$,

where α = coefficient of expansion of air = ·00367.

$$\therefore \quad Vt = \sqrt{\frac{\gamma P(1 + \alpha t)}{\rho_0}} = V_0 \sqrt{1 + \alpha t}.$$

when t is very small

$Vt = V_0 \left(1 + \frac{\alpha t}{2}\right)$ If we take $V_0 = 33200$ cms per econd, and $\alpha = \frac{1}{273}$ then $Vt = 33200 \left(1 + \frac{t}{546}\right)$

$$= 33200 + 61t.$$

That is, the velocity in a gas increases by about 61 cms per degree rise in temperature.

V^2 varies as the absolute temperature.

Let T=absolute temperature $= \frac{1+\alpha t}{\alpha}$

Then $\sqrt{1+\alpha t}=\sqrt{T\alpha}$

$\therefore V t = V_0\sqrt{T\alpha}$, But $V_0\sqrt{\alpha}$=constant

$\therefore Vt \propto \sqrt{T}$

Moisture. Presence of moisture changes V, since density of aqueous vapour is slightly less than that of air, and in addition γ for aqueous vapour is 1·31 whereas that of air is 1·41

Let Vmt=velocity in moist air under pressure P and temperature $t°C$.

Vdt=velocity in dry air under pressure 760 *mm*. and temperature $t°C$.

Then we have $Vmt=\sqrt{\frac{\gamma P}{\rho m}}$

$$Vdt=\sqrt{\frac{\gamma 760}{\rho d}}$$

But ρ_m=weight of I *C. C.* of moist air at pressure P and temperature $t°C$.

=weight of I *C. C.* of dry air at $(P-e)$ and $t°C$+weight of I *C. C.* of moisture at e and $t°C$ [e=saturation pressure of aqueous vapour at $t°C$.]

Again mass of 1 *C. C.* of aqueous vapour

= $\frac{5}{8}$ the mass of 1 *C. C.* of dry air.

$$\rho_m = \frac{P-e}{760}\rho d + \frac{e}{760}\rho d \times \frac{5}{8}$$

$$= \frac{\rho d}{760}\left(P - e + \frac{5}{8}e\right) = \frac{\rho d}{760}(P - {}^{\cdot}378e)$$

We have $\dfrac{Vdt}{Vmt} = \sqrt{\dfrac{760\rho_m}{P \times \rho d}}$

$$\therefore \quad Vdt = Vmt\sqrt{\frac{760\rho d\,(P - {}^{\cdot}378e)}{P\,760\rho d}} = Vmt\sqrt{\frac{P - {}^{\cdot}378e}{P}}$$

$\cdot$ = velocity in dry air at 760 and $t^{\circ}C$.

48. Velocity of a transverse wave along a stretched string.

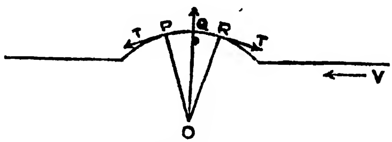


Fig. 33

In fig. 32 PQR represents a transverse wave travelling along a stretched string from left to right with a velocity v. Let us assume that the wave is of a permanent type, so that its from remains unaltered and it moves with a constant velocity. If a velocity V from right to left be now impressed on the string, the wave will remain in one position, for it is moving along the string with a velocity from left to right, while the string is moving in the opposite direction with the same velocity. Again it is found experimentally that the velocity of propagation of a wave along a string does not depend on its shape. We, therefore, assume that part of the wave PQR ...

the crest, at any rate, forms arc of a circle with its centre at O. Let us consider the motion of the element PQR. Since it moves in a circle with a speed v it is acted on by a centrifugal force tending to throw it vertically upwards along OQ. The magnitude of this force

$$=\frac{\text{mass} \times \text{velocity}^2}{r}=\frac{mlv^2}{r}$$

where r=radius of the circle of which PQR is an arc,

m=mass of the string per unit length,

$\frac{l}{2}=PQ=QR.$

But the element is in equilibrium and does not move upwards. It must, therefore, be acted on by an equal force vertically downwords. The only other force acting on the element, however, is that due to a tension T acting tangentially at P and R. The component of each vertically downwards$=T\cos(90^\circ-\theta)=T\sin\theta$;

where $\theta=\angle POQ=\angle QOR.$

Since the element is very small $\frac{l}{2}=PQ=\theta r.$

and $\sin\theta=\theta=\frac{l}{2r}.$

$\therefore$ The total downward force$=2T\sin\theta=2T\theta=\frac{Tl}{r}$

For equilibrium we must have

$$\frac{mlv^2}{r}=\frac{Tl}{r}$$

$$=\frac{T}{m} \text{ or } v=\sqrt{\frac{T}{m}}$$

(v is in cms when T is in dynes and m in grams).

CHAPTER VII.

REFLECTION AND REFRACTION OF WAVE MOTION

49. Reflection and refraction of wave motion.

Let us consider a number of perfectly elastic balls of the same mass arranged in a row, followed by another set of lighter balls similarly arranged along the same row. On striking the first ball of the lighter set in the direction of the row, it moves forward and impinges on the second and comes to rest, while the second moves forward and strikes the third, itself coming to rest and so on. The whole process is as if the first ball moved through the second without disturbing it or being itself disturbed. When the disturbance reaches the first of the heavier set of balls, the lighter ball rebounds after striking the heavier one and moves backward. It now strikes its neighbour and sets it into motion and itself comes to rest, and thus a disturbance is set up which moves back in the opposite direction. The heavier ball in its turn takes up some energy of motion from the rebounding ball and moves forward and comes to rest after striking its neighbour, which in its turn transfers its motion to the next ball and so on. A disturbance thus moves along the row of the heavier set of balls with a definite velocity different from that in the lighter set.

The same thing will happen when a wave motion reaches the surface of separation of two media of unequal densities. Part of the wave energy is transmitted through

the second medium as a *refracted wave motion* and part travels back into the first medium as a *reflected wave motion*.

50. Laws of reflection and refraction of Sound.

Light and radiant heat, like sound, are wave motions, and in a homogeneous medium they propagate uniformly in all directions, falling off in intensity according to the inverse square law Sound undergoes reflection and refraction, and the *laws of reflection and refraction of sound are exactly the same as those of light and heat.*

Laws of reflection When sound waves moving through a medium meet an obstacle, they follow the law of elastic bodies; that is, they return upon themselves forming new waves, which appear to start, from a second centre on the other side of the obstacle. *This phenomenon is called reflection of sound.*

Let us consider spherical waves starting from a point source to undergo reflection at a plane surface. In fig. 34. *S* represents a plane reflecting surface at right angles to the plane of the paper. The incident waves in the plane of the paper, starting from point source *A*, are represented by the concentric circles, and the corresponding reflected waves

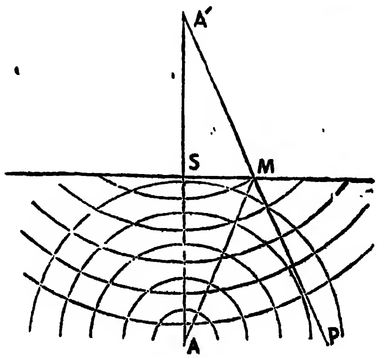


Fig. 34.

are given by the arcs of the concentric circles whose centre

is symmetrically situated behind S at A'. An ear placed at any point P, hears the reflected sound as if it came from A'. AM, a normal to the incident waves is a direct ray, and MP is the corresdonding reflected ray. From the symmetrical positions of A and A' it is clear that the direct or the incident ray and the reflected ray are equally inclined to S and lie in a plane at right angles to S.

Laws of reflection.

1. The angles of incidence and reflection are equal.
2. The incident ray and the reflected ray are in the same plane perpendicular to the reflecting surface

The laws of reflection of sound being the same as those for light and heat, may be demonstrated by similar experimentes.

51. Reflectien from plane and spherical surfaces. In fig. 35 two tubes about 3 feet in length and 1·5 inches in diameter are so adjusted symmetrically with respect to a flat surface S of slate, wood or thick cardboard, that the axes of the tubes are equally inclined to S and lie in a plane at right angles to it. If a watch be held at A its ticks are distinctly heard at B even when a screen C is placed between A and B to cut of off the direct sound of the watch. The hearing is much aided by applying ear to the end of a glass funnel held at B which collects a greater amount of the wave energy. This effect is better demonstrated by

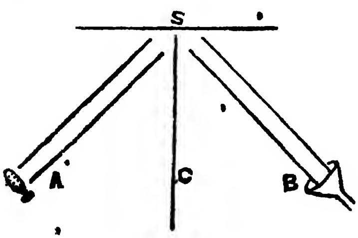


Fig. 35.

placing a whistle at A and a sensitive gas flame at B. S is removed and the pressure of the gas is adjusted till the flame just does not flare. On replacing S in its inital position the flame immediately begins to flare and continues to do so as long as S is in position.

In fig. 36 a watch is held on the axis of a large concave spherical mirror S, and it is found that, when the watch lies between C, the centre of curvature of the mirror and F, its principal focus, the ticking of the watch is heard most distinctly when the funnel is placed at the conjugate point A on the other side of C. As the watch is moved towards F the corresponding position of the funnel moves away from C, and when the watch is at F the sound of the watch is thrown accross a large distance. As in the case of light if a pair of conjugate mirrors be adjusted and a watch be held in the focus of one of them, then its ticking is distinctly heard at the focus of the other even when the mirrors are 10 to 15 yards apart

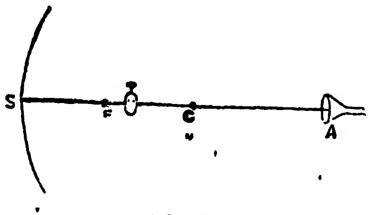


Fig. 36.

52. **Echo** is a familiar example of reflection of sound. We hear echo of our own voice when there is a distant obstacle which can reflect sound directly back to us. The number of syllables that an echo can repeat is proportional to the distance of the obstacle, and it is easily seen that it is not possible to pronounce or hear distinctly more than five syllables in a second. The velocity of sound in air may be taken as 33,000 cms./sec. and therefore sound

travels through 6600 cm. in one fifth of a second. The reflecting surface should be 3300 cm. distant so that sound in going and coming back may take one fifth of a second. The time which elapses between the articulated and the reflected sound is thus one fifth of a second. The two sounds, therefore, do not interfere and the reflected sound is distinctly heard.

At distances less than 3300 cm. the reflected pluse will reach the speaker before a syllable is pronouuced and it will mingle with the voice of the speaker and can not be heard separately.

53. **Multiple echoes**. Sound, like light, may be reflected several times in succession giving rise to a succession of echoes of a sound, and as light under such circumstances gradually falls off in intensity, the echoes also gradually become feebler to the ear. The effect may be due to independent reflections from obstacles at different distances or due to reflections and re-reflections as between two parallel walls. In an old palace of Simonetta in Italy, which forms the sides of a quadrangle an echo repeats from about 30 to 40 times.

54. **Whispering galleries.** A very interesting example of reflection of sound is found in whispering galleries. In the circular gallery of St. Paul's the faintest sound produced on the side of the dome is distinctly heard on the other side, but is not audible in any intermediate position. In fig. 37, S represents a source near the circularr wall of a gallery. The waves diverge in

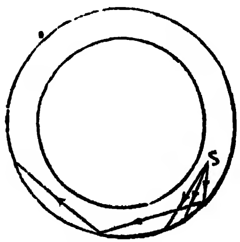


Fig. 37

all directions from S, but all the sound sent out between the circles, as shown in the figure, is reflected round and round the gallery and is thus confined in the air between the wall of the gallery and a cylindrical surface represented by the inner circle, and even a feeble sound S is heard any where near the wall of the gallery in this space.

Sound is also reflected from clouds. It is reflected even in passing through air when it meets the surface of separation of layers of air of different densities and the effect though not strong enough to produce an echo perceptibly weakens the direct sound.

It is found that on some optically clear days sounds cannot be distinctly heard at distance far less than that at which they are clearly heard even during a thick haze. Tyndall explained this effect as being due to the presence of aqueous vapour, which forms an infinite number of very thin striæ, which are quite transparent to light but are opaque to sound, and serve as reflectors of sound waves. Tyndall showed that a sound is much weakened in passing through a medium of alternate layers of light and heavy gases.

It is also found that so long there is continuity of air, sound can pass very easily through the interstices of solids, and it is an interesting fact to find sound passing easily through 10 or 12 layers of dry silk but being effectively stopped by a single layer of wetted silk

55. **Refraction of Sound.** When sound passes from one medium to another in which its velocity is different, the direction of propagation of the waves is in general

altered. *This phenomenon is called refraction of sound.*

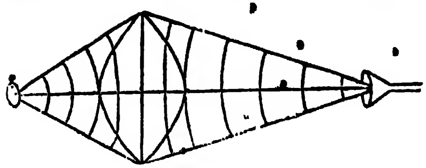


Fig. 38.

The refraction of sound may be demonstrated by using a lens-shaped india-rubber bag filled with any gas whose density is different from that of air. In fig. 38 a convex lens of india-rubber is filled with carbon dioxide in which sound travels more slowly than in air. The sound waves diverging from a point source on the axis of the lens on one side of it are concentrated at a certain point on the axis on the other side. The spherical waves from the source on entering the lens travel more slowly, and the central parts, which reach the lens first and have to pass through more space in the lens, are detained more than those near the edges. The waves inside the lens are thus flattened and on emergence they travel faster again. The edges come out first and gain on the central parts, and thus the waves are curved in the opposite direction and converge near the funnel on the other side.

56. Speaking Tubes. When sound waves pass through a tube, they can not spread out into spheres, as they would do in the open air, but they undergo reflections at the sides of the tubes and are thus confined within the tube. The wave front remains the same in size and the waves move forward with comparatively less decrease in amplitude and therefore in intensity.

57. Wind refraction. Sound is transmitted more effectively when it moves with the wind than against it, and

it is generally found that sound of distant sources which are inaudible in still air are clearly heard when a wind comes from that direction. This is due to the concentration of sound towards the surface of the earth due to the turning of the waves towards the earth by the effect of wind.

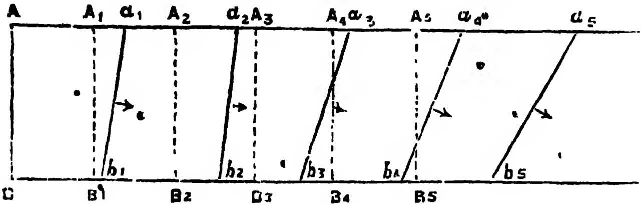


Fig. 39.

In fig. 39. AB represents the section of a wave front set up by a distant source. AB is assumed to be vertical and proceed horizontally. A_1B_1, A_2B_2 etc. represent positions which AB would occupy after equal intervals of time in still air. If wind blows with the sound then the sound is carried faster and the velocity of its propagation is the sum of the velocities of the wind and that of the sound in still air. But the upper layers of air move faster than the lower ones and thus the wave fronts are turned towards the ground and an observer at B_5 has much greater chance of hearing the sound than in still air.

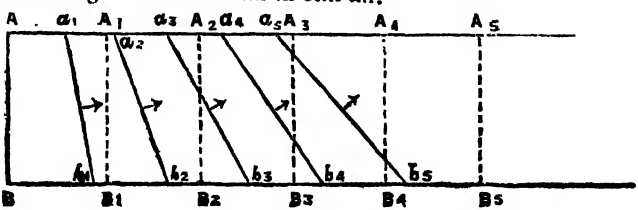


Fig. 40.

In fig 40. the effect of wind blowing in the opposite direction is shown. In this case the velocity of the wind has to be subtracted from that of the sound. The waves near the ground in this case move faster than those in the upper layers and thus the wave fronts are turned upwards, and an observer at B_6 has much less chance of hearing the sound than in still air.

It is a well known interesting fact that sounds are more audible at night than at day time. This effect may be explained as being due to refraction of sound waves on account of variation of temperature of the air. Generally during day the air near ground is hottest and therefore sound moves faster there than in higher levels, with the result that the sound waves are turned upwards as in fig. 40. After dusk the air cools down most rapidly near the ground than in higher levels and is therefore denser. The velocity in denser air being less the waves are turned towwards the ground as in fig. 39, and sounds become audible even at considerable distances

There is another kind of sound refraction, as shown by Reynolds, due to variation of temperature at different levels. For the same temparature the velocity of sound is independent of pressure and would have the same value at all levels. But the temperature generally changes with the level. Again different parts of the earth are unequally heated which produce ascending and desending currents of air of different densities, and sound waves in passing through such layers of air are continually reflected and refracted. They are not necessarily absorbed but are broken up and spread out. Light undergoes similar changes and the effect

is often seen by the dimness of the outlines of distant objects. The long rolling of thunder is probably due, partly at any rate, to the drawing out of a more or less sudden sound of a near flash due to continual reflection and refraction of the waves which eventually break up and spread over a longer distance.

58. Huyghens' principle of secondary waves and envelopes. Let us cosider a point source of disturbance and waves diverging from it in all directions We may concieve the advance of the waves in two ways. We may either assume that the waves start from the source and move onwards without any intermediate consideration, or, may regard the state of things at a certain instant as derivable from that at some previous, instant when the waves are on their way from the source. In an isotropic medium the velocity is the same in all directions and therefore the waves are spherical.

Let S be a point source and AB a section of a spherical wave front deverging from it. The radius of the wave increases continually with the velocity of sound and the disturbance AB will after an instant be at $A'B'$. On Huyghens' principle when the disturbance is at AB each point on AB becomes a centre of disturbance and spherical waves start out from these centres.

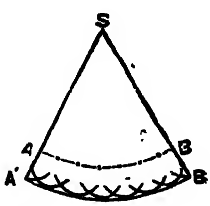


Fig. 41.

The envelope of these *secondary waves* gives the wave front. Huyghens assumed that the effective part of each secondary wave in generating the wave front is confined to that portion of it which touches the envelope.

59. Reflection of a spherical wave from a plane surface.

In fig. 42 a wave front starting from a point source A meets a plane reflecting surface at SS' at some instant.

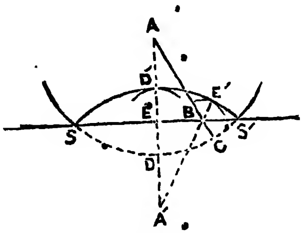


Fig. 42.

Every point of the surface between S and S' becomes a centre of a reflected wave. If the wave had not met any obstacle it would have occupied the position $SD'CS'$ after an instant, but the effect of the reflecting surface is such that when the disturbance has just reached SS', E has become the centre of a reflected spherical wave of radius equal to ED', and any other point, B, for example the centre of a reflected wave of radius equal BE'. All the points have become centres of reflected waves and the envelope of these secondaries is given by the sphere of centre A' and radius equal to $A'D'$ where $AD = A'D'$.

The incident spherical wave starting from A is converted after reflection, into another spherical wave of equal radius with centre at A'. The change undergone on reflection, therefore, is simply to reverse the curvature of the wave.

60 Reflection of a spherical wave at a spherical surface. Let A represent a source and E a concave spherical reflecting surface. The incident wave SS' which would have occupied the position SDS' had there been no

obstacle, is converted into the wave $SD'S'$ on reflection so that $ED = ED'$

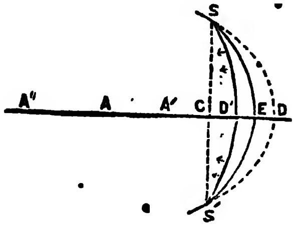


Fig 43.

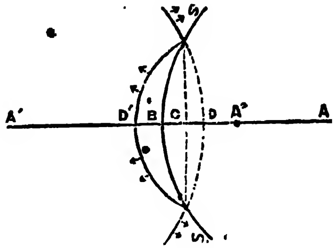


Fig. 44.

From which we have $CD + CD' = 2CE$ or the sum of the curvatures of the incident and the reflected waves is equal to twice the curvature of the reflecting surface.

In fig. 44 the reflecting surface is convex and we have $CD' - CD = 2CB$. Representing curvatures in opposite directions with opposite signs we may state in general that the curvature of the reflected wave is equal to the curvature of the incident wave reversed plus twice the curvature of the reflecting surface.

61. Refraction of a spherical wave at a plane surface. In fig. 45 a spherical wave diverging from a source A falls on a plane surface SS' of a second medium. If the wave had not met the second medium it would have occupied the position SDS' at some instant. The velocity of sound in the second medium is different from that in the first, and E has become

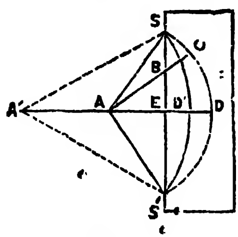


Fig. 45.

the centre of a refracted wave of radius ED' in the second medium where $ED : ED' = v_1 : v_2$ or $ED = \mu ED'$

where v_1 and v_2 are the velocities of sound in the two media, and $\frac{v_1}{v_2}$ = constant $= \mu$. Similarly any other point B has become the centre of a refracted wave of radius equal to BC/μ. The envelope of these secondary waves is given by the wave $SD'S$ whose curvature is μ times the curvature of the incident wave and the effect of refraction has been to change the carvature μ times.

62 Refraction of a spherical wave at a spherical surface.

In fig. 46 a spherical wave diverging from A meets the spherical surface SS' of a second medium Let the velocity of sound in the second medium be less than that in the first. Then the incident wave which would have occupied the position SDS' in an instant had there been no obstacle, gets flattened and occupies the position $SD'S$ so that $ED = \mu ED'$

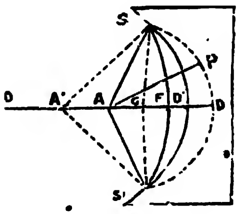


Fig. 46.

The change undergone on refraction, therefore, is to increase or decrease the curvature of the incident wave front, according as the velocity in the second medium is greater or less.

63. Reflection and refraction of a plane wave at a plane surface.

The reflected and the refracted wave fronts set up when

a plane wave reaches the surface of separation of two isotropic media are shown in fig 47. Let SR represent the trace of the incident wave in the plane of the paper. The plane of the incident wave passes through SR perpendicular to the plane of the paper, and the surface of separation of the media is a plane through SS' perpendicular to the plane of the paper.

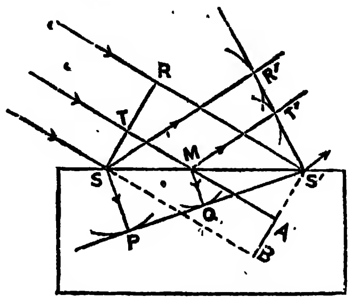


fig 47

Let t = time taken by the wave to advance through RS',

v_1 = velocity of propogation of the wave in the first medium,

v_2 = velocity of propogation of the wave in the second medium.

Describe spheres of radii v_1t and v_2t with S as centre and from S' draw plane $S'T'R'$ and $S'QP$ to touch these spheres perpendicular to the plane of the paper. The tangent plane $S'T'R'$ which is the limit to which the disturbance has advanced in the first medium at the instant when the wave reaches S', represents the reflected wave front. Similarly the tangent plane $S'QP$ touches all the wavelets which started from the points between S and S', and therefore, represents the refracted wave front. SR' and SP, perpendiculars to the reflected and the refracted wave fronts are the reflected ray and the refracted ray respectively. To prove that $S'R'$ touches all other wave fronts in the upper

medium, draw MT' perpendicular to $S'R'$. Then in the right angled triangles $SR'S'$ and SBS', $SR'=SB$ and SS' is common to both. Therefore $\angle SS'R'=\angle SS'B$. Again in the right angled triangles $MT'S'$ and MAS', the angles at T' and A being right angles and $\angle MS'T'=\angle MS'A$, it follows that $\angle SMT'=\angle S'MA$; also MS' is common. Therefore $MT'=MA$

Laws of reflection.

(i) Since $\angle SS'R'=\angle SS'B$, and $\angle SS'B=\angle S'SR$, the incident and reflected wave fronts are equally inclined to the surface SS'. The normals TM and $T'M$ to the incident and reflected wave fronts, and also the normal to the surface SS' are in the plane of the paper, in other words, *the incident ray, the reflected ray and the normal to the surface lat the point of incidence lie in the same plane.*

(ii) Again, since the triangle $SR'S'$ is equal to the triangle SBS' and also the triangle SBS' is equal to the triangle SRS', it follows that $\angle S'SR=\angle SS'R$ and similarly $\angle S'MT'=\angle SMT$, in other words, TM and $T'M$, *the incident and the reflected rays are equally inclined to the surface SS' and therefore to its normal at the point of incidence.*

Again, to prove that $S'QP$ touches all other wave fronts in the lower medium, draw SP and MQ perpendicular to $S'QP$.

Then $\dfrac{SP}{SB}=\dfrac{Vt}{V_0t}=\dfrac{V}{V_0}$, where V_0 and V are the respective velocities of the wave transmission in the upper and the lower media.

Again, since triangles $SS'P$ and $MS'Q$ are similar

$$\frac{MQ}{SP}=\frac{S'M}{S'S} \quad \dots\dots\dots\dots(1)$$

also, since triangles $S'SB$ and $S'MA$ are similar

$$\frac{MA}{S'B}=\frac{S'M}{S'S}=\frac{MQ}{SP} \text{ from } (1)$$

$$\therefore \quad \frac{MQ}{MA}=\frac{SP}{SB}=\frac{V}{V_0}\ldots\ldots\ldots\ldots(2)$$

Laws of refraction :—(i) MQ is at right angles to $S'QP$, which is at right angles to the plane of the paper, therefore, MQ lies in the plane of the paper. *In the words, the incident ray, refracted ray and the normal to the surface at the point of incidence lie in one plane.*

(ii) Let i=angle of incidence of the ray TM.
r=angle of refraction of the ray MQ.

Both of these angles are measured from the normal to the surface SS'. Then $i=(\frac{\pi}{2}-TMS)=MS'A=SS'B$.

Also $r=(\frac{\pi}{2}-QMS')=MS'Q$

$$\sin i=\sin MS'A=\frac{MA}{S'M},$$

$$\sin r=\sin MS'Q=\frac{MQ}{S'M}.$$

$\therefore \quad \dfrac{\sin i}{\sin r}=\dfrac{MA}{MQ}=\dfrac{V_0}{V}=\mu$, the index of refraction from the first to the second medium.

64. Changes undergone on reflection.

Closed end of a tube. Let us consider what happens when a condensation moving through a tube reaches its closed end. The compressed layer of air in trying to expand reacts on the next layer and itself regains its

initial pressure. The second layer thus gets compressed and reacts on the third and so on. Each layer transfers its energy to the next and comes to rest itself. When the condensation reaches the closed end, the compressed layer of air there cannot expand towards this end, and it therefore reacts on the adjacent layer of air behind it, which thus gets compressed and reacts on the next layer and so on. Thus the condensation moves back in the opposite direction through the tube as a condensation. A rarefaction on reaching the closed end similarly gets reflected back as a rarefaction; each pulse as it reaches the end starts one of the same kind back. *The reflected waves, therefore, are copies of the incident waves only with the velocity reversed.*

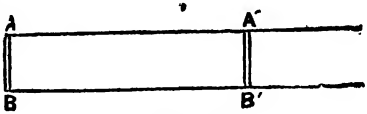


Fig. 48.

Open end of a tube. When the condensation reaches the open end of a tube the pressure in front of it does not increase so fast as when it moves inside the tube. To follow this clearly let us imagine a condensation moving through a uniform tube of unit cross section. Let the disturbance advance inside the tube through a distance l cm. in an infinitely small time t. At the end of this interval the condensation affects l cubic centimetres of air in its front to which its energy is communicated. The velocity of propagation of the disturbance is constant, and thus when the condensation reaches the open end, it will affect in time t all the air within a radius of l cm. from the end

of the tube, to which it communicates the same amount of energy as in the first case. In fig. 48 a condensation $A'B'$ is assumed to move through the tube from right to left. The increase in pressure to the left of AB is not so rapid as it was to the left of $A'B'$. Thus we see that when a condensation moves through a tube, and each layer of air transfers its energy to the next and comes to rest itself, the condensation AB will not have transferred all its energy communicated to it by the previous layer, when it has moved through the same distance as the other layers inside the tube did. The compressed layer of air AB will, therefore, move further than other layers inside the tube, and so, while it reacts on the air in front and compresses it, it will leave a rarefaction behind it, which will move back along the tube. A rarefaction reaching the open end of the tube will similarly spread a rarefaction through the air beyond the end, and start a condensation back along the tube inside it. *The reflected waves in this case, therefore, are copies of the incident waves with the state of density reversed.*

CHAPTER VIII.

SUPERPOSITION OF WAVES.

TATIONARY WAVES—*(INTERFERENCE—BEATS—COMBINATION TONES—CONCORD AND DISCORD.)

65. **General law of superposition.** When several ystems of waves starting from different sources arrive at a oint simultaneously, the motion of the particle of the edium at that point is the algebraic sum of the motions npressed on it by the different systems. This law of uperposition, however, holds good only when the amplitudes re infinitely small.

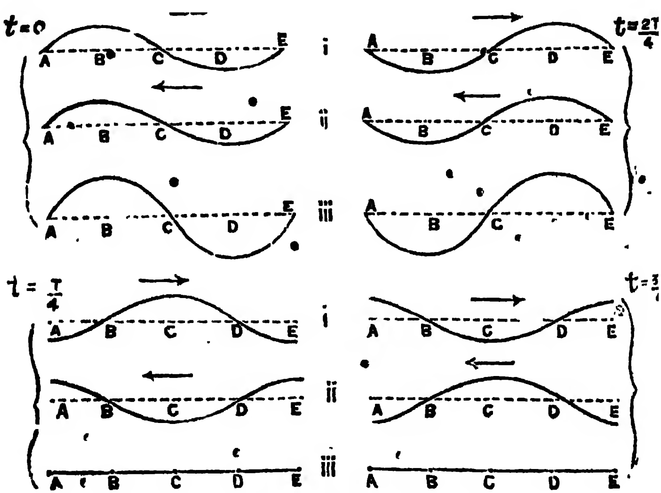
66. **Stationary waves** are formed when two exactly imilar systems of waves move simultaneously through the ame medium in opposite directions with the same velocity.

Case I. When the elementary waves are transverse. Let two exactly similar systems of transverse waves nove through a stretched string simultaneously in opposite irections with the same velocity. Since the waves are noving in opposite directions they will at regular intervals f time agree in phase, that is, the crests and troughs of ne system will exactly coincide with those of the other. et us start our investigation from such an instant, and onfine our attention to the state of things in length *ABCDE* f the string covered by a complete wave.

* Chapters X and XII.

Curves (i) and (ii) represent the displaced positions of the string at some instant due to the elementary waves separately, and the corresponding curve (iii) represents the resultant displaced position of the string due to the combined effect of both.

Fig. 49.



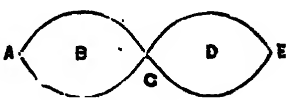


Fig. 50.

At time $t=0$ the displacement of any particle of the string due to both systems is in the same direction and to the same extent, and therefore, its resultant displacement is double the value of either. *A*, *C* and *E* are at rest due to

both. B is displaced through its maximum upwards due to each, and therefore it undergoes double the maximum displacement upwards. Similarly D undergoes double the maximum displacement downwards. Other particles undergo intermediate double displacements upwards or downwards and the string takes up the position represented by curve (iii) $t=o$.

After $T/4$ both systems of waves have advanced through $\lambda/4$ in opposite directions and the relative displacement between them is therefore $\lambda/2$. The displacement of any particle due to both systems being equal and opposite, all the particles are momentarily at rest at this instant as shown by curve (iii) $t=T/4$. [T=time period, λ=wave length.]

After $2T/4$, that is, another interval of $T/4$, the waves have further advanced through $\lambda/4$, and the position taken up by the string is represented by curve (iii) $t=2T/4$.

After $3T/4$ the particles are again momentarily at rest. After $4T/4$ the state of things is the same as at $t=o$, and at successive intervals of $T/4$, the successive changes considered above are repeated. By comparing curves (iii) we find that the points A, C and E, at regular intervals of $\lambda/2$, are permanently at rest and are called *nodes*. Points B and D, halfways between the nodes, are displaced through the maximum in opposite directions alternately at right angles to the length of the string, and are called *antinodes* or *loops*. Other particles undergo intermediate displacements in opposite directions alternately. On acount of persistence of vision we can not distinguish the different stages of motion of the string, and it exhibits the appearance of a flattened ribon as shown in fig, 50.

Case II. When the elementary waves are longitudinal. Let us start from the instant when the maximum compressions and rarefactions of both systems exactly coincide. At this instant the effects of compression and rarefaction due to both systems being the same, both in kind and in degree at every point, the particles undergo double compressions and rarefactions everywhere. Again we know that the particles are displaced along the line of advance of the waves during compression and in the opposite direction during rarefaction, and therefore the effects as regards velocity being equal and opposite at every point due to the two systems, the particles are momentarily at rest all over.

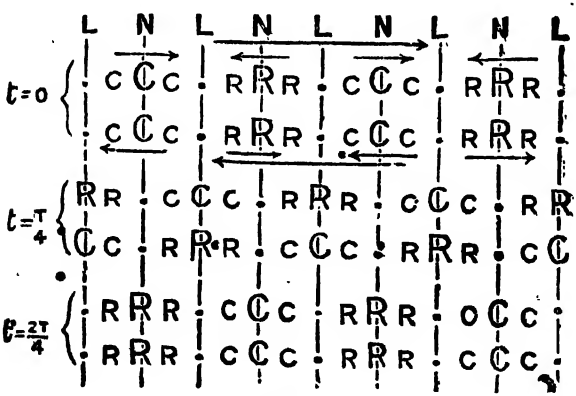


Fig. 51.

After $T/4$ each system has advanced through $\lambda/4$, and the maximum compressions of one system have coincided with the maximum rarefactions of the other. The change in density, therefore, is nil everywhere and the velocity double the value of either.

After $2T/4$ the same effects as at $t=0$ are repeated, with the only difference that the places of maximum changes in density of the two systems are interchanged, each system having advanced through $\lambda/2$ in the opposite direction from its initial position.

After successive intervals of $T/4$, these changes will be repeated. We thus find that there are points at regular intervals of $\lambda/2$, which are alternately places of maximum compression and maximum rarefaction, and where the velocity is constantly zero. These points are the *nodes*. At points half ways between the nodes, the change in density is constantly zero but the velocity constantly double the value of either. Those points are the *antinodes*.

Features of stationary waves. (1) The wave form, that is, the arrangement of the particles at any instant, is in the form of a sine curve (or may be represented by a conventional sine curve in the case of longitudinal waves.)

(2) There are paricles at regular intervals of $\lambda/2$ which are permanently at rest. Other particles execute simple harmonic motions of the same period at right angles (or along the string in the case of the longitudinal waves) to the length of the string, the amplitude increasing continuously from zero to maximum from a node to the next antinode, and then decreasing from the maximum to zero from the antinode to the next node. The phase of all the particles between two consecutive nodes is the same.

3. The vibration of each particle is peculiar to itself, and the wave form, therefore, does not move onwards. The ordinates of the particles undergo proportional changes in value continuously with time, from the positive maximum

through zero to negative maximum, and then back to the positive maximum through zero in T, with the result that the wave form shrinks to a straight line twice in T and alternately expands in opposite directions.

4. At any instant the wave form represented by $ABCDE$ repeats itself at intervals of λ, and therefore the displacement, velocity and acceleration of any particle is the same as those of others at intervals of λ along the string at the same instant.

5. The arrangement at any instant repeats itself at intervals of T.

Analytical treatment. Let the two systems of waves be represented by

$$y_1 = a \sin\frac{2\pi}{\lambda}(Vt - x) \ldots (1)$$

$$y_2 = a \sin\frac{2\pi}{\lambda}(Vt + x) \ldots (2)$$

The result of their combination is given by

$$y = y_1 + y_2 = a \sin\frac{2\pi}{\lambda}(Vt - x) + a \sin\frac{2\pi}{\lambda}(Vt + x)$$

$$= 2a \sin\frac{2\pi Vt}{\lambda} \cos\frac{2\pi x}{\lambda} \ldots (3)$$

From this we have

$$\frac{dy}{dt} = \frac{4\pi a V}{\lambda} \cos\frac{2\pi Vt}{\lambda} \cos\frac{2\pi x}{\lambda}$$

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \sin\frac{2\pi Vt}{\lambda} \sin\frac{2\pi x}{\lambda}.$$

where y = displacement of a particle,

$\frac{dy}{dt}$ = its velocity, $\frac{dy}{dx}$ = its change in density.

(1) At any instant at points along x, the line of advance of the waves, given by $\sin\frac{2\pi x}{\lambda}=0$ or $\cos\frac{2\pi x}{\lambda}=\pm 1$

$y=$maximum, $\frac{dy}{dt}=$maximum, and $\frac{dy}{dx}=$zero.

These points are the *antinodes* and are obtained by putting $\frac{2\pi x}{\lambda}=m\pi$, or $x=\frac{m\lambda}{2}$, where m is any positive integer or zero.

(2) Similarly at the same instant at points given by

$$\cos\frac{2\pi x}{\lambda}=0 \text{ or } \sin\frac{2\pi x}{\lambda}=\pm 1$$

we have $y=$zero, $\frac{dy}{dt}=$zero, and $\frac{dy}{dx}=$maximum.

These points are the *nodes* and are given by

$$\frac{2\pi x}{\lambda}=m\pi+\frac{\pi}{2}, \text{ or } x=\frac{m\lambda}{2}+\frac{\lambda}{4}.$$

(3) Again at times given by

$$\sin\frac{2\pi Vt}{\lambda}=0 \text{ or } \cos\frac{2\pi Vt}{\lambda}=\pm 1$$

we have $y=$zero, $\frac{dy}{dt}=$maximum, and $\frac{dy}{dx}=$zero.

These instants are obtained by putting $\frac{2\pi Vt}{\lambda}=m\pi$

$$\text{or } \frac{2\pi Vt}{VT}=m\pi, \quad \therefore t=\frac{mT}{2}. \quad [\because VT=\lambda]$$

(4) Similarly at times given by

$$\cos\frac{2\pi Vt}{\lambda}=0 \text{ or } \sin\frac{2\pi Vt}{\lambda}=\pm 1$$

$$y=\text{maximum},\ \frac{dy}{dt}=\text{zero},\ \text{and}\ \frac{dy}{dx}=\text{maximum}.$$

These instants are obtained by putting

$$\frac{2\pi Vt}{\lambda}=m\pi+\frac{\pi}{2} \text{ or } t=\frac{mT}{2}+\frac{T}{4}.$$

67. Stationary waves formed by reflection from a wall.

When sound waves from a shrill whistle fall normally on a reflecting surface, the reflected and the incident waves move in opposite directions through the same path in air with the same velocity, and they being similar set up stationary waves. Nodes are formed at regular intervals of $\lambda/2$ from the reflecting surface; and half ways between the nodes antinodes are formed. If a sensitive flame be moved along the normal to the reflecting surface, that is, along the line of advance of the waves, it is found to remain unaffected at the nodes. Then as it is moved along, it begins to flare and at the antinodes the flaring is maximum. The distance between two consecutive nodes gives $\lambda/2$, half the wave length of the sound waves, and from the known value of V the velocity of sound in air, the frequency of the waves is determined from the relation $n\lambda=V$.

A sensitive flame detects a sound whose frequency far exceeds the upper limit of audition and this method

therefore, enables us to determine experimentally the frequency of sounds even when they are inaudiable to us.

68. Resonance. It is found in some cases that when two bodies have the same natural period of vibration, one is set vibrating by the vibrations of the other. This general phenomenon is called *resonance.* In sound the note emitted by a vibrating source is greatly intensified, when the source is held near the open end of an empty vessel of definite shape and size with its natural period of vibration the same as that of the source.

Open tube. The sound emitted by an excited tuning fork is found to be greatly intensified when the fork is held at the mouth of a tube of proper length. If the tube is open at the far end, this happens when the length of the tube is half the wave length of the sound given out by the fork. To follow this let us start with a condensation just given out by the fork and moving down through the tube. On reaching the far end (it is reflected upwards as a rarefaction, which on coming to the near end, is reflected again downwards with a second reversal in form which restores it to its initial form.) A pulse thus has got to move through the tube twice, once down and once up, to be restored to its initial form. If the time taken by the pulse for this excursion be equal to the time period of the fork, then the next pluse from the fork will exactly concur with the reflected pluse, and their amplitudes will be added and the intensity increased four times. Since a pluse undergoes a very large number of reflections before its energy is dissipated away, a large number of amplitudes are added and the sound is greatly intensified.

Let V=velocity of sound in air,

l=length of the tube,

T=time period of the fork.

Then the time taken by a pulse to move through the tube twice$=\frac{2l}{V}$.

If this time be equal to T

then $\frac{2l}{V}=T$ or $2l=VT=\lambda$ or $l=\frac{\lambda}{2}$.

Closed tube. When the tube is closed at the far end a pulse will be reflected from the closed end unchanged in form, and therefore it will have to move through the tube four times, twice down and twice up, to undergo two* reversals in form at the open end to be restored to its initial form. In this case resonance will happen when

$$\frac{4l}{V}=T \text{ or } 4l=VT=\lambda \text{ or } l=\lambda/4.$$

These are the shortest lengths of the tubes which give resonance in the two cases. In each case reflected waves flow continually from both ends in opposite directions and set up stationary waves inside the tubes. There is always a *node* at the closed end and an *antinode* at the open end of a tube, and the shortest distance between a node and an antinode is $\lambda/4$.

As shown in fig. 52, in the case of close tubes resonance will also take place when the lengths of the tubes are odd multipies of $\lambda/4$, and in the case of open tubes as shown

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* See Chapter XI.

in fig. 53, there will be resonance when the lengths of the tubes are even multiples of $\lambda/4$.

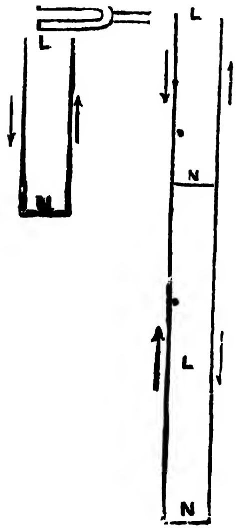


fig. 52.

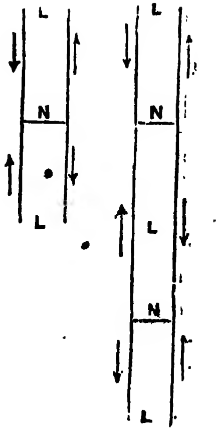


fig. 53.

69. Wind instruments—Organ pipes, and Reed pipes. A wind instrument is generally made of a cylindrical metal tube or a wooden tube of square section. The sounding body in this case is the enclosed column of air inside the tube, one end of which is either closed or open, and the other end has an arrangement for setting up and maintaining the vibrations of this air column. The pitch of the note emitted thus depends entirely on the frequency of vibration of the air column. The meterial of the tube has no effect on the fundamental tone, which is the same for tubes of different meterials. With tubes of different materials,

however, the harmonics are found to be different which make the compound notes different in quality. There are two distinct ways of setting up and maintaining the vibrations of the air column, and the instrument is accordingly called *organ pipe* or a *reed pipe.*

Organ pipe. As shown in fig. 54, when air is blown into the wind chest *a*, it issues from the narrow slit as a stream of thin sheet of air, which breaks against the bevelled lip *b* of the tube. The shock thus encountered causes the air to issue from the slit in an intermittent manner; and the impulses produced by this intermittent flow of air set up vibrations of the air column *c* inside the tube. By properly adjusting the air current when the period of the pulse agrees with the natural period of vibration of the air column, there is resonance and the tube *speaks* powerfully.

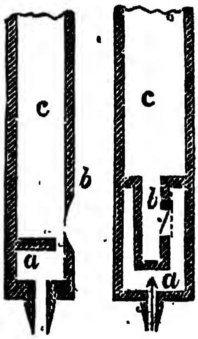


Fig. 54. Fig. 55.

Reed pipe. In this case as shown in fig. 55. air blown into the wind chest *a* in trying to escape sets a thin metal piece *b*, called the reed, into transverse vibrations. The vibrating reed, alternately closes and opens the air passsage and the impulses thus caused by the succession of puffs of air, set up vibrations of the air column *c* inside the tube.

Waves are continually reflected from both ends of a pipe and flow in opposite directions through the tube, sett-

ing up stationary waves in it. There is a node at the closed end of a stopped pipe, and an antinode at the open end of an open pipe, and in both cases there is an antinode at the end where the wind enters, which is always open to some extent.

The position of the nodes and the antinodes are shown by the method devised by Hopkins. For this purpose the front of the tube is made of glass, so that a thin membrane stretched over a cardboard ring and suspended by a string inside the tube, is distinctly seen. Fine sand is strewn on the membrane which is gradually lowered by the string inside the tube, and it is found that when the tube speaks, the sand particles are violently agitated at the antinodes whereas at the nodes they are perfectly unagitated.

Fig. 56.

Another method of fixing the position of the nodes and the antinodes in a pipe was devised by Koenig and is known as *monometric capsule* or *flame*.

A circular hole is made in the side of the pipe and covered by a thin india rubber membrane. A piece of wood or metal in the form of a capsule is fitted into the hole so that it leaves a small space between it and the membrane. In fig. 57. three such capsules provided with small burners are shown fitted to the side of an organ tube. A hollow chamber AB, attached to the side of the tube is filled with coal gas by a rubber tube. The gas comes out of the chamber through three little bent tubes and passes into the space between the

membranes and the capsules, and then out of the burners where it is ignited giving three small flames. Any vibration of the air inside the tube throws the membrance in contact with it into similar state of vibration, which causes corresponding variation in the pressure of the gas in the capsule and thus varies the size of the flame. The vibraions of air inside the tube being very rapid the variations in the size of the flame cannot be distinguished when the flame is seen directly, on account of persistence of vision. To render them distinct a mirror, which can be rotated about a vertical axis is held in front of the flames. When the capsule is at an antinode the flame remains steady, and on rotating the mirror its image appears as a continuous band of light. At a node the flame undergoes rapid changes in size and its image seen in the rotating mirror presents a broken up toothed appearance.

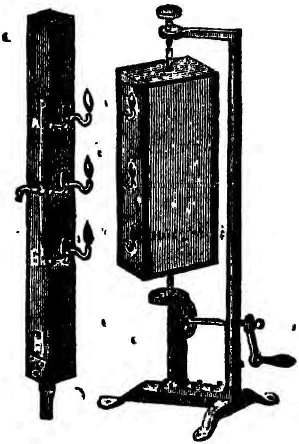


Fig. 57.

70. **Forced vibrations and Resonance.** Let us take a pendulum of period one second at rest, and let a series of blows be applied to it at regular intervals of $1\frac{1}{10}$ seconds. The first blow sets the pendulum into motion. The second blow is applied when the pendulnm has completed a little more than one oscillation, and acts in the direction in which the pendulum is moving and thus increases the amplitude. The effect of the third blow is similar. At the fourth blow

the pendulum is at its position of maximum displacement, and the fifth blow acts while the pendulum is moving in the opposite direction and thus tends to check the motion of the pendulum. The effects of the sixth and seventh blows are similar, and at the eighth blow the pendulum is brought to rest again. The effects of the succeeding blows will be to repeat the whole operation. Let us now imagine that the period of the blow is gradually reduced from $1\frac{1}{18}$ seconds to 1 sec. As an effect of this the number of blows applied to the pendulum before it reaches its extreme position, increases more and more, and therefore, the amplitude becomes greater and greater, and, when the period is reduced to 1 sec. the blow always acts as the pendulum passes its position of rest in the same direction and thus tends always to increase the amplitude.

Here we have assumed so far that the application of the blows has no effect in changing the period of the pendulum. The blows, unless they act when the pendulum passess its position of rest in the same direction, tend to alter the period to some extent. We thus see that a body may be set into periodic motion by the application of a periodic force which changes both in direction and in magnitude. When the natural period of vibration of the body is precisely equal to the period of the force, the body is thrown into powerful vibrations and it is *said to respond to the periodic force.* If however the period of the force differs slightly from the natural period of the body, it begins to vibrate and generally with a small amplitude but period the same as that of the force. The vibrations thus set up in a body are called *forced vibrations.*

71. Forced vibrations set up in pendulums suspended from the same support.

In fig. 58 four pendulums A, A', B and C are suspended from a light horizontal support, the plane of oscillations of the pendulums being perpendicular to the plane of the paper. A and A' are of the same length and therefore of the same period; C is a little smaller and B a little longer than A. A is provided with a rather heavy ball. Starting with all the pendulums at rest, set A into vibration. This causes a periodic force to act on the support at the point of suspension of A, and it is thrown into forced transverse vibrations of the same period as that of A. The vibrations of the support cause a periodic force of the same period as that of A to act at the points of suspention of A', B and C. The natural period of A' being the same as that of the force, it is readily thrown into vibration. The vibrations of B and C are at first intermittent; they take up a little swing, then come to rest and repeat this for some time and finally their intermittent natural vibrations die away and they settle down into a a steady vibration of small amplitude but period precisely the same as that of A.

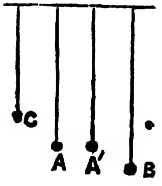


fig. 58.

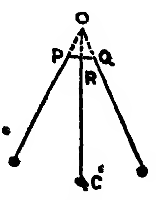


Fig. 59

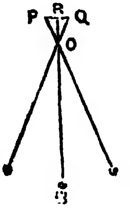


Fig. 60

The mode of vibration of C, is shown in (fig. 59.) R, the point of support moves to and fro along PQ and the period of this motion which the same as that of A, is longer than that of C. C behaves as if its point of support had been at O, which is so situated that a pendulum of length OC would have the same period as that of A. The mode of vibration of B whose natural period is longer than that of the to and fro motion of the support is shown in fig. 60, the string passing through O, so that a pendulum of length OB would have the same period as that of A. It is also evident from the figures that while A, A' and C are in the same phase, B is in the opposite phase.

Analytical treatment Let us now consider a general case. Let a mass m execute a $S.\ H\ M.$ of period T, under the action of a controlling force, such as gravity in the case of a pendulum bob or elastic forces as in the case of a mass attached to a spring.

Then when the displacement of m is x, the acceleration acting on $m = -\left(\frac{2\pi}{T}\right)^2 x$, and the outward force acting on it$= -m\left(\frac{2\pi}{T}\right)^2 x$. (Art. 24.) Now let a harmonically changing force $P \sin \frac{2\pi}{T_1} t$ be applied to m.

m now settles down into a steady vibration, so that the nutural force and the periodic force together give a force required to make m vibrate with a certain amplitude in time T_1, the period of the applied force. Therefore when the displacement of m is x, the acceleration acting on m $= -\left(\frac{2\pi}{T_1}\right)^2 x$ and the outward force acting on it$= -m\left(\frac{2\pi}{T^1}\right)^2 x$.

This is the resultant of the natural force and the applied force. This requires

$$-m\left(\frac{2\pi}{T_1}\right)^2 x = -m\left(\frac{2\pi}{T}\right)^2 x + P \sin \frac{2\pi}{T_1} t$$

or $P \sin \frac{2\pi}{T_1} t = 4\pi^2 m x \left(\frac{1}{T^2} - \frac{1}{T_1^{\,2}}\right) \therefore x = \dfrac{P \sin \frac{2\pi}{T_1} t}{4\pi^2 m. \left(\frac{1}{T^2} - \frac{1}{T_1^{\,2}}\right)}.$

Case 1. when $T_1 > T$ that is, the period of the applied force is greater than the natural period, x is posstive or the forced motion agrees in phase with that of the applied force as shown in fig. 59.

Case 2. When $T_1 < T$, that is, the period of the applied force is less than the natural period, x is negative or the forced motion is opposite in phase to that of the applied force as shown in fig. 60.

Case 3. When $T_1 = T$, that is, $\left(\frac{1}{T^2} - \frac{1}{T_1^{\,2}}\right) = 0$, x is infinite. This means that the displacement will become infinite in this case. In practice however the displacement goes on increasing until the loss of energy due to radiation and dissipation is balanced by the energy supplied to the vibrating body by the applied force. This is the case of *resonance*.

When two clocks which keep nearly the same time are placed on the same stand, the vibrations of their pendulums are communicated to the stand, and thus each pendulum causes a periodic force to act on the other, as a result of which the faster pendulum begins to vibrate a little more slowly and the slower pendulum a little more

rapidly, till finally both the pendulums vibrate similarly in exactly the same period. The vibrations in each case are forced and the period of vibration differs very slightly from the natural period.

A suspension iron bridge at Angers was once thrown into violent vibrations so as to endanger its stability when a regiment was marching over it. The steps of the regiment had a period which coincided with the natural period of the bridge. In the same manner, a ship on the trough of the sea is in a critical position when its natural period of vibration is the same as that of the waves.

72. **Sounding board.** The sound given out by a vibrating body is greatly intensified when it is attached to a hollow wooden box. The vibrations of the sounding body are communicated to the entire mass of the box and the air within it, which thus become the real vibrating source and throws large masses of air surrounding it into vibration. The amount of motion communicated by a vibrating string to the air is generally too small to be perceptible as sound even at a small distance, but when the string is mounted on a sound-board, as in a violin, the sound at once becomes audible even at a great distance.

Since the energy required to set the sound-board into vibration is derived from the vibrating source, and the increase in intensity of the sound, when the vibrating sound-board is present, means more energy being transferred to the surrounding air, it is clear that the duration of sound is proportionately decreased when the sound-board is present.

CHAPTER IX.

VIBRATIONS OF STRINGS AND RODS.

73. Vibrations of stretched strings. In sound *string* means a perfectly elastic and uniform cord, wire or filament of some material stretched between two points. In practice however any real string will have some rigidity, but the effects due to this are negligible if the diameter of the string be very small compared with its length. The vibrations of the string may be *longitudinal or transverse* according as its particles move to and fro about their mean positions along the length of the string or at right angles to it.

74. Transverse vibration. Sonometer. A string may be set into transverse vibrations by bowing, as in a violin, by stretching, as in a piano, or by plucking, as in a harp. The laws of transverse vibrations of a stretched string are generally investigated with the help of a *Sonometer or Monochord.* It consists of a metal wire stretched across two bridges on the top of a hollow wooden sounding box.

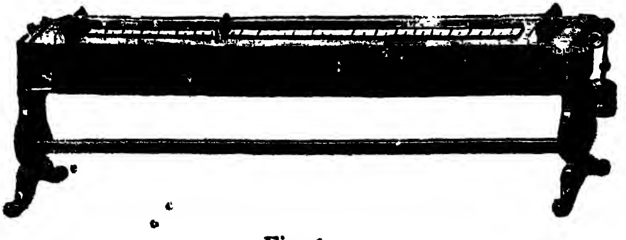


Fig. 61.

One end of the string is fixed and the other passes over a small pulley supporting a scale pan, by placing weight on which the tension acting on the string is suitably adjusted. There is a third movable bridge by means of which the length of the vibrating string can be altered.

75. Reflection of transverse waves. Let a pulse in the form of a crest on one side of a string travel along it from left to right as shown in fig. 62. The particles of the string are drawn towards this side as the crest reaches them, and come back to their normal positions as it leaves them

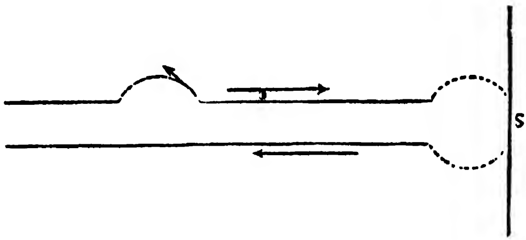


Fig. 62.

and passes on. When it reaches *S*, one of the fixed ends of the string, it can not draw the fixed support to one side and the extra resistance thus encountered causes a rebound, so that the pulse is thrown over on the other side of the string, and starts as a reversed pulse moving back along the string from right to left. *The reflected pulse in this case is a copy of the direct pulse with its form reversed.* A crest is reflected back as a trough and a trough is reflected back as a crest. It is interesting to note in this connection that in the case of water waves, when a crest meets

a solid obstacle it is reflected as a crest, and a trough is reflected as a trough. Similarly when sound waves meet solid obstacle a condensation is reflected as a condensation and a rarefaction as a rarefaction.

76 Stationary waves set up in the string. If instead of a single wave a train of waves travels along the string, then on account of reflection at the fixed ends, two similar systems of waves will move through the string in opposite directions with the same velocity, and will set up stationary waves, with nodes at the fixed ends. The string may vibrate as a whole or it may divide itself into any number of equal parts, separated by nodes, each part vibrating as an independent string. When the string is bowed at the centre, it vibrates as a whole as shown in fig. 63 (i) and is said to give out the *fundamental* note.

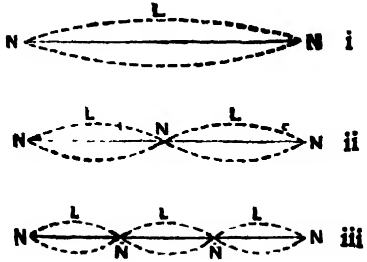


fig. 63.

If the centre is damped and one half is bowed, the other half also vibrates as shown in fig 63 (ii) and the string gives out the *octave*, with double the frequency of the fundamental. Again if the string is damped at a point which cuts off one

third of its length, and the shorter length is bowed, it vibrates as shown in fig 63 (iii) giving out the *twelfth* with three times the frequency of the fundamental and so on. The series of notes obtained when the string is divided into its aliquot parts are called the *harmonics* of the fundamental note.

77. Laws of transverse vibrations of a stretched string.

We have seen that the velocity of propagation of a transverse pulse along a stretched string is given by

$$v=\sqrt{\frac{T}{m}}\ldots\ldots(\text{i})$$

where v=velocity of propagation of the pulse,

T=tension with which the string is stretched

$=W\times g$ units of force [W is the load stretching the wire, and g the acceleration due to gravity.]

m=mass of the string per unit length,

$=\pi r^2\rho$ [r is radius of the wire, and ρ the density of the meterial of the string.]

Let l=length of the string between two consecutive nodes$=\lambda/2$

n=frequency of vibration of the string,

The since $v=n\lambda=2nl$

we have $v=2nl \quad \sqrt{\frac{T}{m}}$

or $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$

$$=\frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}}\ldots\ldots(\text{ii})$$

Since n is expressed in terms of four variables l, r, T and ρ, there are four laws of vibration.

1. *Law of length.* From (ii) we find that when r, T and ρ remain contant $n \propto \frac{1}{l}$, that is, for wires of the same material and radius and stretched by the same force, the frequencies of their vibration are inversely proportional to their lengths.

2. *Law of radius.* When l, T and ρ remain constant $n \propto \frac{1}{r}$, that is, for wires of the same length and material, and stretched by the same force, the frequencies of their vibration are inversely proportional to their radii.

3. *Law of tension.* When l, r and ρ remain constant $n \propto \sqrt{T}$, that is, for wires of the same length, radius and material the frequencies of their vibration are directly proportional to the square roots of the tensions with which they are stretched.

4. *Law of density.* When l, r and T remain constant $n \propto \frac{1}{\sqrt{\rho}}$, that is, for wires of the same length and radius but of different materials and stretched by the same force, the frequencies of their vibration are inversely proportional to the square roots of their densities

78. Experimental verification of the laws. To prove the first law the wire of the sonometer is made to vibrate as a whole and the frequency of its vibration determined. Let l be the length of the string and n its frequency of vibration. Next the length of the

vibrating string is suitably altered by the sliding bridge and the frequency is determined. When the lengths are $l/2$, $l/3$, $l/4$ etc. it is found that the frequencies are $2n$, $3n$, $4n$ etc. respectively, which proves the first law.

It will be considered in a subsequent chapter how the frequency of vibration of the string in experimentally determined.

To prove the second law wires of the same material but of different diameters are mounted on the same sonometer, so that their vibrating lengths are the same. They are stretched by the same load and it is found that when their diameters are as 1 : 2, the frequencies of their vibration are as 2 : 1. If the diameters have any other values d_1 and d_2, the frequencies are as $d_2 : d_1$, which proves the second law.

To prove the third law two identical wires (of the same material and radius) are mounted on the sonometer, so that the vibrating lengths are equal. They are stretched with different loads and the frequencies of their vibration are determined, and it is found that when the loads are as 4 : 9, the frequencies are as 2 : 3. If the loads have any other values W_1 and W_2, the frequencies are as $\sqrt{W_1} : \sqrt{W_2}$ which proves the third law.

In this case the same wire may be stretched with different loads and the frequency of vibration determined for each load. The frequencies will be found to be proportional to the square roots of the loads.

To prove the fourth law two wires of the same radius but of different materials having densities ρ_1 and ρ_2 are mounted on the sonometer, and they are stretched

by the same load. The frequencies of their vibration are separately determined and it is found that they are inversely proportional to the densities. The same result is established in another way. The length of one string is kept unaltered and that of the other is changed till the two strings vibrate in unison. In this case it is found that

$\frac{l_1}{l_2}=\frac{\sqrt{\rho_1}}{\sqrt{\rho_2}}$; where l_1 and l_2 are the lengths giving unison.

But we have already seen that $\frac{n_1}{n_2}=\frac{l_2}{l_1}$

$\therefore\ \frac{n_1}{n_2}=\frac{\sqrt{\rho_2}}{\sqrt{\rho_1}}$, which proves the fourth law.

79. **Melde's experiment.** The laws of transverse vibration of a stretched string are very conveniently verified by Melde's experiment. A silk chord or a thin wire is fastened at one end to the prong of a large tuning fork, while the other end passes over a small pulley supporting known weights.

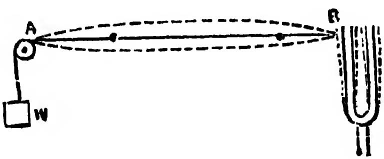


Fig. 64.

Case I. When the prong moves backwards and forwards along the length of the string. In this case the frequency of the string is just half of that of the fork. As shown in fig. 64, starting with the fork at rest, as the prong moves towards its extreme position *B* the string becomes

slack and is allowed to sag. Next as the string moves back and is furthest from B the string is stretched tight and is horizontal. As the string comes back again to B the string is slack again but due to inertia of motion upwards it goes up as shown in the figure. Thus the string executes only half of the vibration while the fork completes one vibration.

Case II. When the prong vibrates at right angles to the length of the string. Let us turn the fork through 90°, so that the prong moves to and fro at right angles to the length of the string, and swings the string in unison with it. Thus the prong and the string have the same frequency.

Let n = frequency of the fork. Then in the second case when the load on the pan is such that the tension T on the string satisfies the relation $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$, the string vibrates as a whole most energetically. When the tension is changed to $T/4$, the string again vibrates very energetically but in two segments, with a node in the middle and with double the frequency of the fundamental. For tensions $T/9$, $T/16$ etc., the vibrating segments are 3, 4 etc. in number, with 2, 3 etc. nodes between the fixed ends and having frequencies three times, four times etc. the frequency of the fundamental.

80. Longitudinal vibration of strings. In this case the particles of the string move backwards and forwards parallel to the length of the string whose frequency of vibration is independent of the stretching force or the tension with which the string is stretched. When a particle of the

string is displaced from its normal position, the restoring force which tends to bring the particle back to its normal position, is brought into play by the stress caused by the displacement, and is thus independent of any previously existent stress which affects all the particles of the string equally. The velocity with which a longitudinal wave travels along a string, therefore, is independent of the tension and depends only on the elasticity and density of the material.

$$v=\sqrt{\frac{E}{\rho}}\dots\dots\dots(1)$$

When both ends of the string are fixed and the string gives out its fundamental, there are two nodes at the fixed ends and a loop in the middle. In this case

$$l=\text{length of the string}=\lambda/2$$

$$\therefore\quad v=n\lambda=2nl=\sqrt{\frac{E}{\rho}}.$$

$$\text{or } n=\frac{l}{2l}\sqrt{\frac{E}{\rho}}\dots\dots\dots(\text{ii})$$

E = Young's modulus of the material of the string,

ρ = its density.

E should in strictness be the adiabatic value and not that obtained by any of the statical methods which gives the isothermal modulus. But the difference between the two values is very small and may be neglected.

81. Comparison of the velocities of propagation of transverse and longitudinal waves along a stretched string.

We have seen that velocity of propagation of a transverse wave along a stretched string $= V_t=\sqrt{\frac{T}{m}}$

and V_l = velocity of propagation of a longitudinal wave along a stretched string $= \sqrt{\frac{E}{\rho}}$.

But m = mass of the string per unit length,
= cross section of the wire × its density,
$= s\rho$. [s = cross section and ρ = density]

$$\therefore \quad V_t = \sqrt{\frac{T}{\rho s}} \quad \ldots\ldots(\mathrm{i})$$

$$V_l = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{Es}{\rho s}} \quad \ldots\ldots(\mathrm{ii})$$

Thus in order that V_t may be equal to V_l the tension acting on the string must be equal to Es. But Es is the force which will stretch the wire to double its length. We see, therefore, that longitudinal waves in wires travel much faster than any transverse ones.

82. Superposition of harmonic curves. Fourier's theorem. If we construct a curve such that the ordinate at any point is the algebraic sum of the ordinates of two or more curves at the same point, then this curve gives the result of superposition of the elementary curves.

When the components are two harmonic curves of the same period, then the result of their superposition is a third harmonic curve of the same period.

Again we know that a periodic curve repeats itself at intervals of λ; and therefore if two or more harmonic curves have periods, each some aliquot part of a given period, then their superposition will give a new periodic curve of the given period.

Conversely we see that any periodic curve which may be represented as a time-distance graph, may be taken to be made up of periodic curves whose periods form aliquot

parts of the period of the given curve. The general statement of this result is known as *Fourier's theorem.*

83. **Fourier's theorem** may be interpreted to show "that the composition of commensurate simple harmonic motions of suitable amplitudes and phases is competent to produce a finite periodic motion of any form whatever," that is, "any periodic curve of period λ, which is continuous, always at a finite distance from the axis, and with only one ordinate for each abscissa, may be formed by the superposition of harmonic curves of wave-lengths λ, $\lambda/2$, $\lambda/3$ etc. by properly adjusting the epochs and amplitudes of the constituents, and there is only one set of harmonics which will thus form it."

84. **Quality of a musical sound. Note. Tone. Overtones. Harmonics.** In Art. 11 it is stated that the quality of a musical sound depends on the mode of vibration of the source at each instant and thus on the form of the waves set up. We have seen that a string can vibrate as a whole or in any number of equal parts. It is not found possible to set a string into vibration as a whole without causing to some extent its subdivision. Thus when a string vibrates, it gives out generally a mixture of the fundamental with the higher tones. The frequencies of the higher tones in the case of the string are exact multiples of that of the fundamental. But in the case of a tuning fork, rod or any other sounding body where the restoring force of vibration is due to the rigidity of the material, the frequencies of the constituents of the note are not exact multiples of the lowest in the series. The sound given out by a source is generally composite in character, being a mixture of the fundamental with the higher tones.

This mixture is different in different sounding bodies and is the cause of that characteristic feature of sound which is called *quality* or *character*.

The compound sound is called a *note.* It is made up of constituents which are themselves irresoluble and are called *tones*. The lowest one in the series, called the fundamental, is prominent and others called the *overtones* or *upper partials* of the fundamental are feeble, so that the pitch assigned to the note is only that of the fundamental. When the frequencies of the overtones are exact multiples of that of the fundamental they are called *harmonics*.

85. Harmonic vibrations detected by resonance. Mount a second string on the sonometer and let it be tuned to unison with one of the harmonics of the first string. When the first string vibrates, it gives out in addition to the fundamental tone, some of its harmonics. If the harmonic to which the second string has been tuned be present in the vibrations of the first string, then the second string takes up energy from the first and is itself thrown into vibration. It does not vibrate, however, if this particular harmonic be not present in the vibrations of the first string.

86 Analysis of compound sounds by resonance. Resonator. A resonater consists of a hollow metallic vessel, the large volume of air inside which vibrates in unison with a particular note. The resonance globe of Helmholtz consists of a hollow sphere with two openings, one of which *b* is small and is turned towards the origin of sound, and

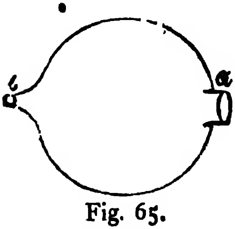


Fig. 65.

the other *a* which is wide is applied to the ear through an india rubber tube. The air inside the globe has a natural period of vibration of its own, which depends on the diameter of the sphere and that of the orifice. When the note proper to the resonance globe is present among the harmonics of the compound note sounded, it is reinforced by resonance and is thereby rendered more powerful than other components and becomes distinctly audible. Thus by means of a series of such globes, the whole series of harmonics of a compound note may be isolated and studied.

87. The ear and hearing. The human ear consists of several complicated structures, and a full description of it lies outside the province of Physics and will be found on recent works on Anatomy and Physiology. In this treatise we shall simply consider the physical facts with which we are concerned.

The human ear may be divided into three parts; the external ear which includes the *pinna* or *auricle* and *meatus* or external opening; the middle ear or drum; and the internal ear or *labyrinth*. The middle ear is a cavity separated from the external ear by the *tympanic membrane* and has a chain of three small bones or oscicles, which connect this membrane with the internal ear. The principal part of the internal ear where the fibres of the auditory nerve terminate is the membranous labyrinth, a complicated structure of sacs and tubes filled with a fluid and held in a cavity, called the bony labyrinth, in the periotic bone. The bony labyrinth consists of a central cavity into which three semicircular canals and the canal of the *cochlea* open. The vestibular portion of the labyrinth has two

sacs, connected by a narrow tube ; three membranous semi-circular canals open into the former sac and the latter is connected with a membranous tube in the *cochlea* which contains the organ of *Corti*.

88. Helmholtz's theory of audition. Probable existence of a series of a resonators in the ear.

When sound waves enter the cavity of the ear, they are concentrated upon the tympanic membrane and set it into vibration ; its vibrations are transmitted through a chain of bones in the middle ear to the internal ear, where they cause certain delicate structures of the organ of *Corti* and the *basilar membrane* to excite the fibres of the auditory nerve to transmit sound impulses to the brain.

The basilar membrane is lined with a very large number of fine fibres which are the terminations of the auditory nerve. This membrane is loose longitudinally and tense radially and may be considered to consist of a number of parallel strings as in a harp. Holmholtz assumed that these fibres act as resonators, each fibre being tuned to a definite frequency, so that the series of fibres would respond to vibrations of all the frequencies throughout the range of audition. Thus when the disturbance produced by a note reaches the fluid in the *Cochlea*, it throws into sympathetic vibration just those fibres whose natural periods agree with those of the simple tones present in the note. Each fibre responds to a particular frequency and so breaks up the note into its constituents. According to this view, therefore, the ear is a kind of practical Fourier s theorem.

Or at any rate, it may be assumed that a simple tone when it reaches the ear, excites a select group of fibres ; the middle members of the group having exact agreement are excited strongly, and the extreme members whose periods nearly agree with that of the tone are very feebly excited. When two simple tones of nearly the same frequency reach the ear simultaneously, the two groups of fibres that are excited overlap, in other words, some fibres are common to both, and their vibrations being the resultant of two *S H.Ms* of nearly the same period, varies in amplitude from the sum to the difference of the amplitudes of the two components, causing a periodic waxing and waning in the intensity of the sound which are perceived as beats. (Art 99) When the beats are sufficiently slow they are perceived distinctly as separate maxima of sound. If, however, the beats are so rapid that the auditory nerve cannot recover completely between the two stimuli the effect is a discord. Again when the beats are very rapid, they link themselves together like the periodic impulses of an ordinary musical note, and thus escape perception owing to persistence of impressions.

89. Vibration of solid rods.

Longitudinal vibrations Longitudinal waves consisting of alternate compressions and rarefactions travel along rods of elastic solids as along air in tubes. They are reflected from the fixed ends of a rod as at closed ends of a tube, and at free ends of a rod as at open ends of a tube. Stationary waves are thus set up in a rod as in a tube.

The general expression for the velocity of propagation of longitudinal waves $V = \sqrt{\frac{E}{\rho}}$ holds good for any

elastic medium and therefore it gives the velocity of such waves in rods of elastic solids.

Case I. When the rod is clamped in the middle, there is a node at the fixed point and two loops at the free ends, and its length l satisfies the relations

$$2l = \lambda_1 = 3\lambda_2 = 5\lambda_3 = \text{etc.}$$

according as the rod gives out its fundamental or the first, second etc. overtones The frequencies of the fundamental and the overtones are as 1 : 3 : 5 etc. and their wave lengths λ_1, λ_2, λ_3, etc.

Case II When the rod is fixed at one end, there is a node at the fixed end and a loop at the free end, and the length of the rod satisfies the relations

$$4l = \lambda_1 = 3\lambda_2 = 5\lambda_3 \text{ etc.}$$

for the fundamental and the overtones.

Transverse vibrations. Transverse waves travel along rods of elastic solids as along a stretched string, but the rod need not be stretched, since it is not the tension but the rigidity (elasticity of shape) of the solid, that brings the restoring force into play, when the particles of the rod are displaced from their normal positions. In the case of the string we neclected its rigidity and assumed that the restoring force was due to tension only. In the case of the rod, however, tension has no effect and the restitution is entirely due to the rigidity of the material.

When the transverse waves are harmonic the velocity of propagation is found to be

$\propto t$, the thickness in the direction of the displacement,

$\propto \frac{1}{\lambda}$, ($\lambda$=wave length)

$\propto \sqrt{\frac{Y}{\rho}}$, Y=Young's modulus for the material and ρ its density.

Non-periodic transverse waves are found to have no definite velocity, and since the velocity depends on λ, the laws of transverse vibrations of rods are very much complicated and a full treatment of the subject is beyond the scope of this work.

1. *Effect of clamping.* When the rod is clamped at one end and vibrates in its fundamental form, there is a node at the fixed end and a loop at the free end as shown in fig. 66 (i). The modes of vibration when the rod sounds its first and second overtones are shown in figs. 66 (ii) and (iii).

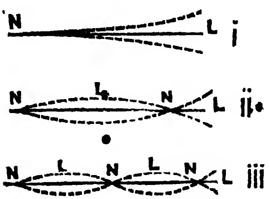


Fig. 66

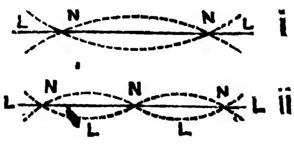


Fig. 67

The frequencies of the overtones are not exact multiples of that of the fundamental in this case. The frequencies of the fundamental and the first, second, third etc. overtones are as 1 : 5·29 : 8·27 : 10·21 etc. and therefore the overtones are not the harmonics of the fundamental.

2. *Effect of fixed supports.* If the rod instead of being clamped is allowed merely to rest on fixed supports at the nodes as shown in fig. 67 and prolonged towards the clamp with a free end, it will vibrate in its fundamental form as shown in fig. 67 (i). The frequencies of the fundamental and its overtones in this case are as 1 : 2·92 : 4·87. etc. The mode of vibration when the rod sounds its first overtone is shown in fig. 67 (ii).

90 Vocal organ in man. Human voice. The vocal organ in man is essentially a double reed instrument. It consists of a wind pipe, *the trachlea*, which leads to the lungs at one end and forms the vibrating part, the *larynx*, at the other. Two cartilagenous membranes, called the *vocal chords*, are stretched across the top of the wind pipe edge to edge with a narrow slit between them, and the two edges of the slit act as reeds. The vocal chords have muscles attached to them by which their tension and the frequency of vibrations can be altered. The edges of the slit are forced apart by the outgoing current of breath, and brought together again by their own elasticity and muscular tension. The breath current is thus broken up into rapid succession of puffs, which produce the vocal sounds. The larynx is longer and the vocal chords are larger and thicker in men than in women and children; the vocal chords in men therefore vibrate more slowly and the sounds produced are lower in pitch.

The sounds given out by the vocal chords are much modified by the cavity of the mouth. The different vowels are produced by changing the shape and size of the mouth cavity in different ways. According to the *relative pitch*

theory the mouth resounds to certain harmonics of the fundamental given out by the vocal chords, so that a sound which contains among its harmonics the same multiples of the fundamental, which may have any frequency, is recognised as the same vowel. According to another theory, called the *fixed pitch* theory a note corresponds to a certain vowel, when one or more other notes of fixed frequencies are added to it. The note thus added, which determines the vowel, is not a harmonic of the note which produces the greater part of the sound. On what the quality of a vowel exactly depends is still a matter of controversy, and generally both the theories are held together, so that a vowel would require for its production certain harmonics of the fundamental and also certain independent notes of fixed pitch. The vowels can be sustained, and in fact they form different ways of beginning and ending a vowel sound and are produced by modifying the vowels by the tooth, tongue and lips.

91. **Phonograph.** This is an instrument devised by Edison for recording and reproducing sound and is similar in principle to the rotating drum arrangement for determining the frequency of vibration as explained in chap. XI. A hollow cylinder covered with wax is so mounted that it can be turned mechanically by a handle fixed to its axis, part of which is a screw working in a nut, so that as the cylinder rotates it also moves endwise. The sound to be recorded is directed into a funnel which converges the waves upon a thin glass diaphram, to the centre of which is attached a small style with a sharp cutting edge. The vibrations of the diaphram cause the style to vibrate longitudinally at right angles to the surface of the cylinder, and thus cut a

groove of variable depth on the wax of the rotating cylinder. This groove thus forms a record of the vibrations of the diaphram. To reproduce the sound the cylinder is rotated back to its original position and the cutting style is replaced by a blunt point. On rotating the cylinder in the same direction and with the same speed as in the previous case, the blunt point is caused to pass along the groove and vibrate exactly in the same way as the recording style, and its motions being communicated to the diaphram, the latter repeats the movements it executed under the influence of the incident sound waves. The vibrating diaphram thus sets up waves which reproduce the original sound.

92 **Vibrations of tuning forks.** We have seen in Art 89 fig. 67 (i) that when a rod free at both ends and supported at two nodes with a loop at the middle, sounds its fundamental, its ends move up and down together. The same thing occurs when the rod vibrates with any even number of nodes and a loop at its centre. If the rod is bent gradually at the middle as shown in fig. 68 the two consecutive nodes *NN* on either side of the centre come nearer together and the transverse motion at the centre, which is a loop becomes less and less. If the rod is held by a stem at the middle, then since very little energy is communicated to the stem, the vibrations of the rod are not interfered to any appreciable extent on this account.

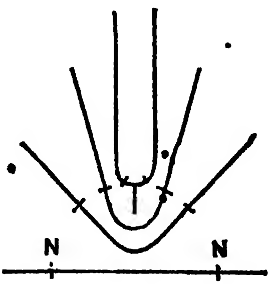


fig. 68

Ultimately when the two limbs of the fork are parallel, the two nodes come so near each other that they practically coincide, and the motion at the centre, which is still a loop, practically vanishes. The stem now takes up no energy from the vibrating rod, which thus vibrates for a long time. The rod thus bent is converted into a tuning fork, and we see how it is that the prongs of a vibrating tuning fork alternately approach and recede from each other. As explained in Art. 89 the overtones are not harmonics of the fundamental The frequencies corresponding to different modes of vibration though in principle derivable from that of a rod free at both ends and supported at two nodes, are practically the same as that of a rod fixed at one end. The overtones are hard to be set up and die out sooner, and thus the fundamental lasts much longer, and the note given out by a fork, which has been excited by gentle bowing and has been vibrating for some time is practically a simple tone. This is the reason why tuning forks are of such importance in sound. A rise in temperature increases the length of the rod and decreases the elasticity of its material, and in the case of steel forks M'Leod and Clarke found that the temperature coefficient of the frequency is given by $n_t = n_0 (1 - \cdot 00011 t)$

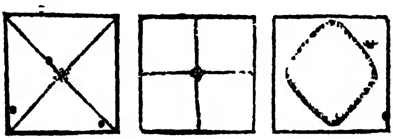
93. Vibration of plates.

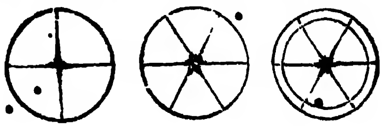
1. Square plates. As shown in fig. 2 when a square plate is fixed at its centre horizontally on a vertical stand and is bowed against its edge, it gives out a note. By damping and exciting the plate in different ways it is thrown into various modes of vibration giving out different notes. Sand particles spread on the upper surface of the

plate are tossed away from the vibrating parts and settle down along nodal lines where the plate is permanently at rest. Fig. 69 (ii) shows the nodal lines when the middle point

Fig. 69.

i ii iii





i ii iii

Fig. 70.

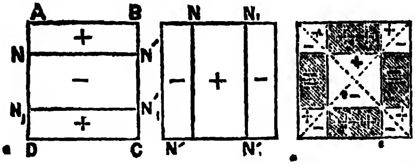
of one of the edges is damped and the bow is drawn across the edge at one of the corners. Fig 69 (i) shows the form when the plate is damped at one corner and bowed in the middle of an edge. Various types of such figures were obtained by Chladni experimentally and are known as *Chladni's Figures.*

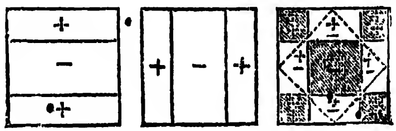
Wheatstone's analysis—To explain the formation of the nodal lines Wheatstone regards the plate to be made up of a set of rods parallel to one side, *AB* (fig. 71 i) with another set of rods parrallel to *AD* superposed on the first set. During the course of transverse vibrations of the plate portions of the plate move up and down the plane of the plate. The upward motions may be represented with a + sign and the downward motions with a – sign. The rods

parallel to AB may be assumed to vibrate with nodes of all of them along NN' and $N_1N'_1$ (Fig. 71 i) and those parallel

Fig. 71.

i ii iii





i ii iii

Fig. 72.

to AD with their nodes along NN' and $N_1N'_1$ (Fig. 71 ii) If the two sets of rods vibrate simultaneously then their resultant motion at any point would give the motion of the plate at that point. Let us now see what happens when the central portions of the two sets of rods are in opposite phase. The result is shown in fig. 71 iii. In the shaded portions the effects are added and in the unshaded portions the effects are neutralised. The nodal lines in this case are the diagonals represented by the dotted lines. When the central portions of the two sets are in the same phase the nodal line is given by the dotted line in fig. 72 iii. The reason why the nodal line obtained experimetally, shown in fig. 69 iii, is a little inwards from the edge, is due to the fact that the amplitude of motion of a rod is not the same all along its length but, starting from a maximum at the ends decreases

gradually towards the centre. This figure is obtained when the plate is free at the centre and is supported at a point on the nodal line. As in the case of a tuning fork the overtones are not harmonics of the fundamental. The vibration frequencies of plates of the same material and shape, vibrating so as to give the same type of nodal lines, are found to be directly proportional to the thickness and inversely proportional to the area of the plates

94. **Circular plates.** A disc like a square plate on being damped and excited differently gives out notes of different pitch. Fig. 70 (i) shows the radial nodal lines when the disc fixed at the centre horizontally on a vertical stand, is touched by the finger nail at a certain point on the edge and is bowed across the edge at a point 45° from the damped point. Since a nodal line seperates two parts of the plate vibrating in opposite phase, there are always an even number of radial nodal lines in such cases. When the disc is free at the centre, nodal lines in the form of circles and other curves are obtained by damping and exciting the plate in various ways. Fig. 70 (iii) corresponds to the case when the disc is clamped on points on a concentric circular nodal line and is bowed through an aperture in the centre. The vibration frequency of a plate, like that of a disc, is proportional to its thickness and inversely proportional to the its area i.e. the square of its radius.

95 **Bells.** We have seen how a rod bent in the middle is converted into a tuning fork. Similarly a bell may be regarded to be a circular plate bent so as to form a concave surface. When a bell is set into vibration by being

bowed or struck, it divides itself into an even number of vibrating sectors separated by nodal lines, the portions of the bell on opposite sides of a nodal line vibrating in opposite phase. The vibrations are both radial and tangential. At the nodal lines the radial motion is zero and the tangential motion is maximum. As shown in fig 73 when the rim on one side of a node is outside the mean position, the rim on the other side is inside the mean position, and thus half of the sectors dialates, and the other half contracts along the radius. To allow for the radial change in length between adjacent nodes a motion of the rim takes place in its own plane, so that at some of the nodes the material of the bell undergoes dialatation while at the others it undergoes compression. The overtones of the bell, like those of the fork and the disc, are not harmonics of the fundamental, and the frequency of vibration of a bell like that of a disc, is proportional to its thickness and inversely proportional to the square of its radius.

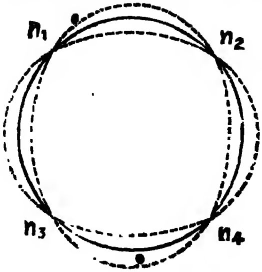


Fig. 73.

96. Electric transmission of sound. Telephone.

This is an instrument devised by Graham Bell for transmitting articulate speech electrically from one place to another. As shown in section in fig. 74 it consists of a steel cylindrical magnet *M*, round one pole of which is fitted a flat coil having a large number of of turns of fine insulated copper wire. The magnet with the coil is encased in wood, and the terminals of the

coil are connected to binding screws *BB*. A diaphram of soft iron, thin as a letter paper, is held close in front of the magnet, by a mouthpiece *E*, which has a central aperture. By some screw arrangement the distance of *S* from the pole

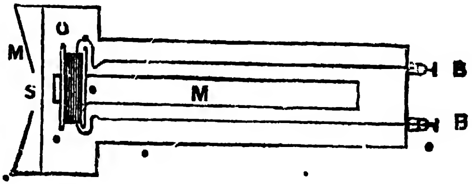


Fig. 74

of the magnet can be altered. The binding screws *BB*, of one instrument are connected by wires with those of another, and it is found that when an operator speaks near the mouthpiece of one instrument his speech is distinctly heard by another operator with his ear placed near the mouthpiece of the other instrument.

The diaphram *S*, gets magnetised by induction and in its turn reacts on the magnet, whose pole strength thus changes differently for different positions of *S*. When sound waves impinge on *S*, and set it into vibration, its distance from the magnetic pole changes continuously, causing corresponding changes in the strength of the magnetic pole. The number of (magnetic) tubes of induction passing through the coil, which depends on the strength of the pole, thus changes, setting up a varying induced electromotive force given by $-\frac{dN}{dt}$ (rate of change of the number of tubes with time). The current set up by the induced electromotive force passes the coil in the second telephone and thereby

changes the pole strength of the magnet in this instrument. The force exerted by the magnetic pole on the diaphram changes, causing it to repeat the same movements as those of the diaphram in the first telephone. The waves set up by the vibrating diaphram thus reproduce the original speech. The resistance of the circuit weakens the strength of the current, and the sound reproduced is considerably enfeebled on this account.

97. **Microphone.** This is an instrument devised by Hughes for reproducing and magnifying sound electrically in a telephone. As shown in fig. 75 it consists of a pencil C of gas carbon, pointed at each end, resting lightly on two supports A, B of the same kind of carbon. A and B are held horizontally in a vertical wooden stand, and are connected to the terminals of a circuit in which a small battery and a telephone are included. Resistance of carbon changes with the pressure acting on it. Thus when sound waves falling on C, set it into motion, the pressure at the points of contact of A and B changes the resistance of the circuit. The current in the circuit accordingly changes, causing the disc of the telephone to vibrate and reproduce the sound. The arrangement is so very sensitive and the magnification of sound so great that even the disturbance produced by a fly walking on the base of the instrument is heard with surprising loudness at a considerable distance in the telephone.

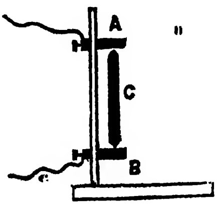


Fig. 75.

CHAPTER X.

INTERFERENCE—BEATS—DOPPLER'S EFFECT—EFFECT OF OBSTACLES—SHADOWS.

98. Superposition of two trains of waves. Interference.

It follows from the general law of superposition that when two trains of waves move through the same medium simultaneously, either in the same direction or in opposite directions, the actual motion of each particle of the medium is the resultant of the motions due to each system separately. In the case of sound waves if the condensations of one system coincide with the condensations of the other and the rarefactions with the rarefactions, their amplitudes are added, and the sound produced by such coincidence is louder than that due to either system separately. But when the condensations of one system coincide with the rarefactions of the other, the resultant amplitude is the difference of the amplitudes of the two systems and therefore the sounds mutually weaken each other. If the amplitudes are equal and the waves agree in phase then the intensity of the resultant sound is four times the intensity of either, and when they are opposite in phase there is complete silence. This phenomenon of coalescence and destruction of effect is called *interference.*

Analytical treatment. Let the two systems of waves be represented by

$$y_1 = a \sin(\theta - e_1),$$

$$\text{and} \quad y_2 = b \sin(\theta - e_2).$$

The result of their superposition is given by

$$y = y_1 + y_2 = a \sin(\theta - e_1) + b \sin(\theta - e_2)$$
$$= C \sin(\theta - E)$$

$$\text{where } C^2 = a^2 + b^2 + 2\,ab \cos(e_1 - e_2)$$

$$\text{and } \tan E = \frac{a \sin e_1 + b \sin e_2}{a \cos e_1 + b \cos e_2} \quad \text{Art. 31.}$$

C, the amplitude of the resultant wave varies between $a+b$ and $a-b$ according to the value of $(e_1 - e_2)$ Thus in this case the result of the superposition is a third wave of the same period and of ampiltude depending on the relative phase difference.

The interference of sound waves is demonstrated by using the instrument shown in section in fig 76. A vibrating

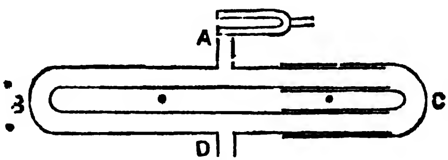


Fig. 76

tuning fork is held at the open end A of a tube which divides into two branches B and C and unites again and ends in a common tube D. The end of the branch C slides over its remaining part so that sound waves can be caused to travel different distances through the two branches. If the paths through B and C are equal, the waves on coming out of D, agree in phase and strengthen each other, and a

sensitive flame placed near D flares. If the sliding portion of C is pulled out gradually till the two paths differ by $\lambda/2$, so that the waves on emergence oppose in phase, there is complete silence and the flame remains unaffected now. The flaring of the flame will be maximum when the path difference is any even multiple of $\lambda/2$, and therefore the waves agree in phase, and there will be no effect when the path difference is any odd multiple of $\lambda/2$, and therefore the waves are opposite in phase.

99 Superposition of two trains of waves of nearly the same frequency. Beats.

When two sounds of nearly the same ~~same~~ frequency are produced together and their waves reach the ear simultaneously, we do not generally distinguish them, but perceive a single note having a periodic waxing and waning in intensity called beats. The periodic changes in the intensity are due to the fact that at certain equal intervals the waves agree in phase and strengthen each other and half ways between they are opposite in phase and weaken each other. The waves in combination, therefore, give a series of points of alternately maximum and minimum intensity which advance with the velocity of sound and are perceived as beats. For example if the waves are of frequencies 200 and 204 and wave lengths λ and λ' respectively, then we have $200\lambda = 204\lambda'$ or $50\lambda = 51\lambda'$. Thus if the waves agree in phase at a certain point, they will again do so at intervals of 50λ or $51\lambda'$ and halfways between they will oppose. Since the time taken by sound to move through 50λ or $51\lambda'$ is $\frac{50}{200}$ or $\frac{51}{204}$ or $\frac{1}{4}$ sec. the number of

beats per second is four, that is *the difference of the frequencies of the primary waves.*

Analytical treatment.

Let two trains of waves of the same amplitude a, and frequencies n_1 and n_2 be represented by equations (i) and (ii).

$$y_1 = a \sin 2\pi n_1 t \ldots\ldots \text{(i)}$$

$$y_2 = a \sin 2\pi n_2 t \ldots\ldots \text{(ii)}$$

The result of their superposition is given by

$$y = y_1 + y_2 = a \sin 2\pi n_1 t + a \sin 2\pi n_2 t$$

$$= 2a\cos \frac{2\pi(n_1 - n_2)t}{2} \sin \frac{2\pi(n_1 + n_2)t}{2}$$

This may be taken to represent a periodic vibration of amplitude $2a\cos\frac{2\pi(n_1 - n_2)t}{2}$ and frequency $\frac{n_1 + n_2}{2}$, midway between the frequencies of the two trains. The amplitude varies from $2a$ through 0 to $-2a$. Since the intensity is protionnal to the square of the amplitude, the interval between successive maxima and zero values is $\frac{1}{n_1 - n_2}$ sec, and the frequency of the beats is $n_1 - n_2$, *which is the difference of the frequencies of the two waves.*

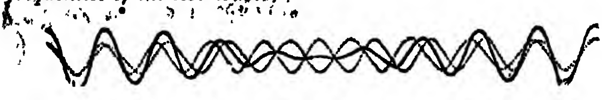


Fig. 77

Graphical representation. In fig. 77 the dotted lines represent two systems of waves of frequencies 9 and 10

respectively, and the thick line the result of their superposition. The amplitude in the resultant curve, as shown in the figure, alternately passes through maximum and minimum values, the maxima occuring when the component waves agree in phase, and the minima when they oppose. Let λ and λ' be the wave lengths. Then in this case $9\lambda = 10\lambda'$. Thus if the waves agree in phase at a certain point, they will again do so at intervals of 9λ or $10\lambda'$.

100. Doppler's effect. The apparent change in the pitch of a note with the motion of the source, observer or medium is called the *Doppler's effect* and the principle which gives an explanation of this effect is called the *Doppler's principle.*

We have seen that the velocity of sound depends only on the elasticity and the density of the medium through which it propagates. It follows from this that the velocity of a sound wave relalively to the air, when once it is given out, is the same whether the source is at rest or in motion relatively to the earth.

Source at rest. Let the source be at rest giving out n waves per second in still air. The waves will advance with the velocity of sound V, and thus a length V having n waves in it will pass an observer at rest in one second.

Source in motion. If the source moves towards the observer, it will follow up the waves previously given out by it, and thus a greater number of waves will be crowded in a given length of air than if the source had been at rest. The number of waves in length V which will pass the observer at rest in one second being greater than n in this case, the apparent pitch as perceived by the observer is

higher. On the other hand if the source moves away from the observer the number of waves in a given length of air is smaller and therefore the apparent pitch is lower.

Observer in motion. When the observer approaches the source, he picks up an extra number of waves occupying the length moved through by him in addition to the waves he would have received had he been at rest. The apparent pitch is thus higher. On the other hand when he recedes from the source he loses a number of waves occupying the the length through which he moves and the apparent pitch is lower.

The reasons underlying the changes in pitch in the two cases—motion of the source and motion of the observer—are entirely different. When the source is in motion the number of waves in a given length changes, that is, the *wave length is altered.* Whereas when the observer is in motion the wave length remains the same, but the observer either loses or gains a certain number of waves.

Wind. The effect of wind is simply to change the value of V relatively to the earth (V = velocity of sound relatively to the air). If w be the velocity of the wind relatively to the earth, then the sound waves advance with velocity $V+w$ when the wind blows with it, and its velocity is $V-w$ when the wind blows in the opposite direction.

Fig. 78.

As shown in fig. 78

Let V=velocity of sound relatively to the air,

w=velocity of the wind relatively to the earth,

v_s=velocity of the source relatively to the earth,

and v_0=velocity of the observer relatively to the earth.

Let the motions of the source, observer and the wind be from left to right as shown in the figure. The waves given out by the source at S_1 at a certain instant moves to Q_1 in one second, where $S_1Q_1=V+w$.

The source has given out n waves and has itself moved to S_2 in one second. Thus n waves have been crowded in the length $S_2Q_1=V+w-v_s$; and the new wave length therefore $=\dfrac{V+w-v_s}{n}$

Again the wave which reaches the observer at O_1, at a certain instant moves to Q_2 in one second; where $O_1Q_2=V+w$. The observer in the meantime has moved to O_2, so that the length of the wave that has passed him in one second $=V+w-v_0$. The number of waves in this length $=\dfrac{V+w-v_0}{\text{wave length}}=n\dfrac{V+w-v_0}{V+w-v_s}=n'$=pitch of the sound as perceived by the observer.

If the motion of the source, observer or the wind be not in the direction from the source to the observer, then we have to take the components of their motion along this direction in the above calculation.

Case I. When $v_0=v_s$, that is, the source and the observer are either at rest or have the same velocity in the same direction $n'=n$.

Case II. When $v_s > v_o$, that is the source and the observer approach, $n' > n$. Similarly when $v_o > v_s$ that is the source and the observer recede, $n' < n$. It is to be noted here that n' in this case depends on the velocities of the source and the observer and not simply on the rate of approach $v_s - v_o$.

Case III. When v_o and v_s are very small compared with $V+w$.

$\dfrac{(V+w)-v_o}{(V+w)-v_s} = 1 + \dfrac{v_s - v_o}{V+w}$ approximately, and therefore a motion of the source has the same effect as an equal and opposite motion of the observer, but the reasons underlying the changes in pitch are different in the two cases as considered above.

Case IV. When v_o and v_s are not small compared with V, motion of the source does not produce the same effect as an equal and opposite motion of the observer.

We have $n' = n \dfrac{V+w-v_o}{V+w-v_s}$

Let $w=0$, then when $v_o=0$ and $v_s = V/3$

Changing the sign of v_s we get

$$n' = n \frac{V}{V+V/3} = \frac{3}{4} n.$$

Again let $w=0$, then when $v_s=0$, and $v_o = V/3$

$$n' = n \frac{V-V/3}{V} = \frac{2}{3} n.$$

Thus the frequencies are different in the two cases.

Graphical representation.

Case I. $V > v_s$ In fig. 79 let S_1, S_2, S_3, S_4 etc. represent the positions of a point source after equal intervals of time,

that is, o, t, $2t$, $3t$ etc. in sill air. Draw circles of radii $3tV$, $2tV$, tV with centres S_1, S_2, and S_3 respectively.

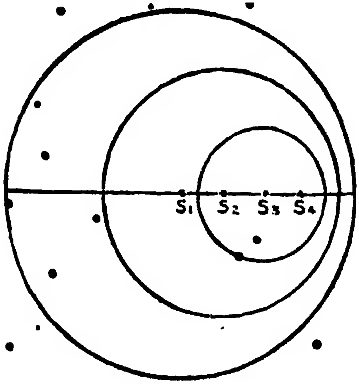


Fig. 79.

Since a wave, when once it is produced, moves with the same velocity whether the source is at rest or in motion, it follows that these circles are the limits to which the disturbance set up by the moving source at S_1, S_2, S_3, have advanced in the plane of the paper at the instant when the source just comes to S_4, and are therefore the sections of the wave fronts in the plane of the paper. In this case the wave front due to the disturbance produced at a certain instant lies entirely outside all the wave fronts due to disturbances set up at all subsequent times, and as a result of this the waves crowd together in the direction of motion of the source and thin away in the opposite direction.

Case II. $V < v_s$. In this case, as shown in fig. 80 the wave fronts due to disturbances set up at subsequent times lie partly or wholly outside the wave fronts due to distur-

bances set up previously. As explained in Art 58, the effective part of each wave in generating the resultant wave front is confined to that part of it which touches the envelope. The tangent plane S_4R, is thus a branch of the resultant wave front on one side of S_1S_4. A similar tangent plane on the other side of S_1S_4 is another branch of the resultant wave front. The elementary waves strengthen one another along these tangent planes and interfere at other points. The two branches of the wave front are inclined at a constant angle 2θ, where θ=angle which either wave front makes with S_1S_4.

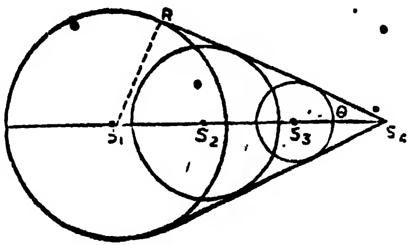


Fig. 80.

Since $S_1R=4tV$
and $S_1S_4=4tv_s$

$$\sin\theta=\frac{S_1R}{S_1S_4}=\frac{4tV}{4tv_s}=\frac{V}{v_s}.$$

V being constant θ decreases as v increases. By measuring θ, v is found out from the known value of V.

Dopper's effect is verified in a simple way in the laboratory by tightly fitting a whistle of high pitch into a stout rubber tubing, the free end of which is placed either in the

mouth or fitted on the bellows. The whistle is blown steadily and whirled in a circle. To an observer at some distance away the pitch of the note given out by the whistle is found to rise and fall, the maximum changes in pitch taking place when the whistle is moving towards or away from the observer.

Questions.

1. An engine in cutting between two bridges, is whistling when its velocity is 1/20 that of sound in air. Compare the frequencies of the echoes from the two bridges to an observer between them.

w and v_0 are zero in this case.

Therefore frequency of the note heard on drawing apart

$$n_1 = n\frac{V}{V+v_s} = \frac{20}{21}$$

Frequency of the note on approach

$$n_2 = n\frac{V}{V-v_s} = \frac{20}{19}$$

$$\therefore \quad \frac{n_2}{n_1} = \frac{21}{19} = 1{\cdot}105$$

2. At what speed must two trains run so that the pitch of the whistle of one as heard on the other may change in the ratio 9 : 8 when they cross each other. Velocity of sound in air is 1100 ft./sec.

Let the velocity of the trains $= v$

Frequency of the note on approach $= n_1 = n\dfrac{V+v}{V-v}$

Frequency of the note on drawing apart $= n_2 = n\dfrac{V-v}{V+v}$

Since $n_1 = \frac{9}{8}n_2$ we have $\dfrac{n(V+v)}{V-v} = \dfrac{9}{8}\,n\left(\dfrac{V-v}{V+v}\right)$

$$\text{or} \quad 8(V+v)^2=9(V-v)^2$$

Putting $V=1100$, we get $v=32$ ft./sec.

3. The angle between the two branches of the wave of disturbance made by a bullet passing through air is 60°. If the speed of the wave is 1100 ft. per second, what is the speed of the bullet?

V=Velocity of sound in air $=1100$ ft./sec.

Angle between the two branches of the wave $=2\theta=60°$

$$\therefore \sin\theta=\frac{V}{v_s} \quad \therefore \sin 30° = \frac{1}{2} = \frac{1100}{v_s}$$

$$\text{or } v_s = 2200 \text{ ft./sec.}$$

101. Effect of obstacles. We have seen how to regard light and sound as wave motions. But whereas after passing through a hole in a screen light is seen to propagate in straight lines, sound is seen to spread out in all directions. This rectilinear propogation or bending of the waves depends, as we shall presently see, on their wave lengths.

In fig. 81 MM' is the trace of a plane wave front of light at right angles to the plane of the paper and P a point in front of it. Each of the ether particles in MM' becomes a centre of disturbance. To consider the combined effect of the wavelets from all the particles at the point P, draw PA perpendicular to MM', and let $PA=r$. With

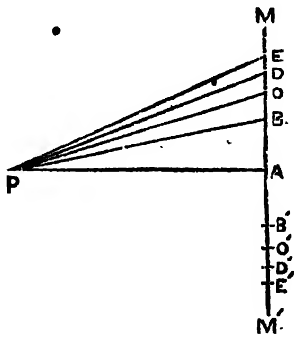


Fig. 81

centre P and radii equal to $r+\lambda/2$, $r+2\lambda/2$, $r+3\lambda/2$ etc. describe a series of circles cutting MM' at B, C, D, etc. forming half-period zones. Join BP, CP, DP etc. Now $BP-AP=\lambda/2$; therefore, the waves from A and B reach P in opposite phases and destroy each other's effect. In the same manner the waves from B and C interfere at P, and for the same reason waves from all the particles between A and B interfere with the waves from all the particles between B and C.

The effect produced at P by the wavelets from all the particles in a zone may be assumed to depend on (i) the area of the zone, which gives the number of ether particles which send out wavelets to P, (ii) the distance of the zone from P and (iii) the inclination of MM' to the line joining P to the zone.

(*i*) *Area of the zones.*

We have $AB^2=PB^2-AP^2=(r+\lambda/2)^2-r^2$

$$=r^2+r\lambda+\frac{\lambda^2}{4}-r^2=r\lambda, \text{ neglecting } \lambda^2, \lambda \text{ being very small.}$$

similarly $AC^2=\left(r+\dfrac{2\lambda}{2}\right)^2-r^2=2r\lambda$

$AD^2=\left(r+\dfrac{3\lambda}{2}\right)^2-r^2=3r\lambda$ and so on

Thus the area of the circle $AB=\pi r\lambda$

$\therefore$ the area of the zone $BC=2\pi r\lambda-\pi r\lambda=\pi r\lambda$

Similarly the area of the zone CD $3\pi r\lambda-2\pi r\lambda=\pi r\lambda$ and so on.

Hence we find that the area of all the zones is the same.

(ii) and (iii) *Distance and inclination of the zones.*

The radius of the nth. zone $=nr\lambda$.

Taking $r=100$ cm. and the wave length of the D line of sodium $=6\times10^{-5}$ cm. the values of AB, AC, AD etc. radii of the consecutive zones starting from the first are found on calculation to be ·077, ·109, ·134, ·154, cm. etc. respectively and their widths ·032, ·025, ·020, ·019 cm. respectively. The width of the tenth zone is. ·014 cm. Thus we find that starting from the pole A the width of the zones falls off at first very quickly and then more slowly, and in the case under consideration, after a distance of about 2 millimeters the zones become so very narrow that the distance of two adjacent zones from P and their inclination with the line joining them to P become practically the same, and the wavelets from them reach P in opposite phases, and therefore, their effects at P become equal and opposite. This means that if an obstacle placed at A is large enough to cut off the first few half period zones, the wavelets from the remaining zones with just cancel each other and no light will be perceived at P, in other words, the obstacle will cast a sharp shadow at P.

102. Difference between properties of light and sound waves. Shadows. As shown in the preceeding article if we take the wave length of the D line of sodium to be 6×10^{-5} cm. and if we suppose that MM' is 100 cm. from the eye, then the radius of the tenth half period zone is equal to $\sqrt{\cdot06}=\cdot1$ cm. On the other hand in the case of sound if we take the wave length of the middle C tone of a pianoforte to be 120 cm. and if we suppose that MM' is 100 cm from the ear, then the radius of the tenth half period zone is equal to $\sqrt{10\times100\times120}$, which is approximately equal to

300 cm. This means that in the case of light an obstacle only 1 mm. in diameter is equivalent to the screening action, in the case of sound, of an obstacle about 3 meters in diameter. The reason, therefore, why in the case of sound waves 'shadows' are so seldom formed is that the wave length of the disturbance is too great compared to the size of the obstacle.

When, however, the pitch of the sound is very high and therefore its wave length small, the screening effect of a comparatively small obstacle can be easily demonstrated. Thus in the case of the shrill whistle of a locomotive engine the shadow cast by a small card held a few centimeters away from the ear is distinctly noticable.

103. Transmission of sound through apertures.

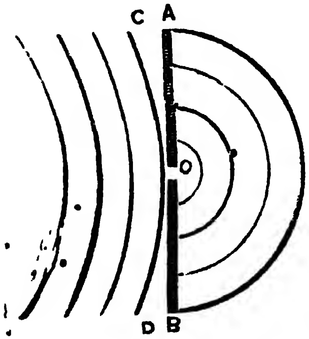


Fig 82.

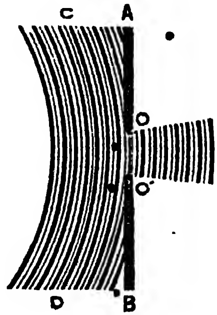


Fig 83

In fig, 82, *AB* represents a screen, of some material which does not transmit sound and *O* is a hole in it. *CD* represents a train of spherical waves consisting of alternate

compressions and rarefactions from a source to the left of *AB*. As a compression reaches the hole, the air in the hole is compressed and moves to the right, and if the wave length of the waves is longer than the diameter of the hole, the state of compression spreads spherically in every direction to the right of the screen, the phenomenon being similar to the effect of a pulse reaching the open and of a tube considered in art. 64. In the figure the thick and the thin lines represent the wave fronts of the maximum compression and maximum rarefaction In this case the sound is audible at any point to the right of the screen. But when the wave length is smaller than the diameter of the hole, as shown in fig. 83 the waves after passing the hole hardly spreads at all but advances at right angles to the wave front.

CHAPTER XI.

Frequency And Velocity

104. Experimental determination of frequency.

Direct and indirect methods. The frequency or the number of vibrations per second of a source emitting a note of definite pitch may be determined *directly* by some mechanical or graphical methods. But when it can not be directly measured then the note given out by a second source, whose frequency of vibration can be adjusted and determined, is brought in unison with the given note. When in unison both notes have the same frequency, and therefore, the frequency of one being known that of the other is known also. The comparison of frequency in such cases is conveniently made by means of *beats*. The frequency of the second note is adjusted till beats are distinctly heard, which enable us without exact perception of pitch to bring the two notes near each other, and to determine their relation Let n be the frequency of the second note and a the frequency of the beats. Then the frequency to be determined $= n \pm a$. The circumstances under which the experiment is made decide whether a is positive or negative.

105. Savart's wheel. It consists of a toothed wheel with equidistant saw teeth, so mounted on an axis passing through its centre, that it can be rotated rapidly. An indicator attached to the side of the instrument gives the number of revolutions of the wheel in a given time. A card

or a thin metal plate is held against the teeth so that while the wheel rotates the card is lifted up and sends out a wave each time a tooth passes it, and the succession of waves thus produced gives a musical note whose pitch increases with the speed of rotation. To determine the frequency of a given note the speed of rotation is adjusted till the note given out by the instrument is in unison with the given note. When in unison both notes have the same frequency.

Let n = number of teeth on the wheel,

m = number of revolutions of the wheel per second.

Then the required frequency = nm.

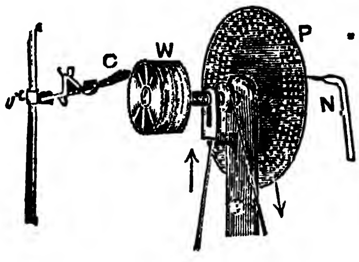


Fig. 84.

106 Seebeck's cardboard siren. It consists of a circular plate of cardboard pierced with concentric circular rows of equidistant holes, mounted like Savart's wheel, so that it can be rotated rapidly. The speed of revolution is determined similarly with the help of an indicator attached to the side of the instrument. A stream of air is directed through a nozzle against one of the circles of holes. As the plate rotates sufficiently rapidly air comes out of the holes in succession of puffs and produces a musical note whose pitch rises with the speed of revolution.

Let n = number of holes in the circle,

m = number of revolutions of the plate per second.

Then the required frequency = nm.

In fig. 84 Savart's wheels *W*, and Seebeck's cardboard siren *P*, are shown together fitted on the same spindle and geared so as to be rotated at any required speed. *C* is a thin metal strip pressed against the teeth of one of the wheels. *N* is a nozzle through which a jet of air is directed to any ring of holes on the cardboard siren.

The spindle is rotated rapidly by a belt gearing it with a large wheel, which is turned by hand. The large wheel is not shown in the figure.

107. Latour's siren. Seebeck's siren was greatly improved by Cagniard-Latour and Helmholtz. The improved type consists of a nearly cylindrical wind box *A*, the top of which is pierced with a circular row of equidistant holes. Another disc with similar circle of holes and provided with a vertical spindle is pivoted so that it rotates freely about the spindle just clearing the upper surface of the wind-box. The holes are not perpendicular to the plates but are inclined in opposite directions as shown in fig. 85. A side tube is attached below to introduce air in the box, and the rotation of the disc is effected when air in escaping through the holes impinge on the sides of the holes in the upper disc and set it into rotation.

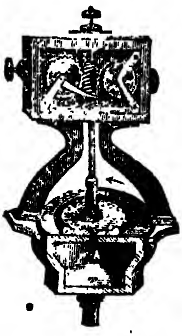


Fig. 85.

During rotation the holes are alternately closed and opened and the air escapes in successive puffs whose frequency depends on the speed of rotation. The upper part of the vertical spindle is an endless screw, so that as it rotates it drives a toothed wheel which in its turn drives another toothed wheel. Two needles attached to the axes of these wheels move round dials and record the speed of rotation of the disc.

To find the frequency of a given note fix the siren on the bellows, and regulate the pressure of wind till the note emitted by the siren is in unison with the given note. Manage to keep the pressure constant for a short time and record the readings of the needles at the beginning and end of a known interval of time.

Let n=number of holes on the disc,

m=number of revolutions of the disc in time t.

Then the required frequency$=\frac{nm}{t}$

108. Stroboscopic disc. The stroboscope consists of a disc, mounted as in the case of Seebeck's siren, so that it can be rapidly rotated and its speed of rotation recorded by an indicator. The disc is pierced with a circle of equidistant holes or rather radial slits at equal intervals. The vibrating body, the frequency of which is to be determined, is held near the apertures on one side of the disc, and a bright point fixed on it is seen through the slits from the other side. As the disc gradually slackens from very high speed the bright point is generally seen as a line of light, and then

when the speed being reduced lies within certain limits, it appears as a number of separate points. When the frequency of the slits is slightly greater than that of the vibrating body a single moving point is seen, which appears stationary when the frequencies are equal. The reason why the point appears stationary is that at regular intervals of *T*, the time period of the vibrating body, it will be passing the same position, and if in this period a slit, moves on and occupies the space occupied by the preceeding slit, then the point will be seen in the same position and owing to persistence of vision it will appear stationary. But it will also appear stationary when its frequency is two times, three times etc. of that of the slits. To get the correct result, therefore, the speed of rotation of the disc should be the highest at which the point appears stationary.

Let m=number of revolutions of the disc per second.

n=number of holes or slits.

Then the frequency of the vibrating body$=mn$.

109 Rotating drum—Graphic method. A hollow cylinder is so mounted that it can be turned mechanically by a handle fixed to its axis, part of which is a screw working in a nut, so that as the cylinder rotates it also moves endwise. Smoked paper is gummed round the cylinder, and a tuning fork is firmly clamped so that it may vibrate parallel to the axis of the cylinder, and a light style attached to one of its prongs may just touch the smoked paper. When the handle is turned and the fork is excited a wavy line is traced on the smoked paper. *To measure the frequency* by this apparatus the tuning fork is insulated and

connected to one terminal of the secondary of an induction coil, the other end of which is connected to the cylinder.

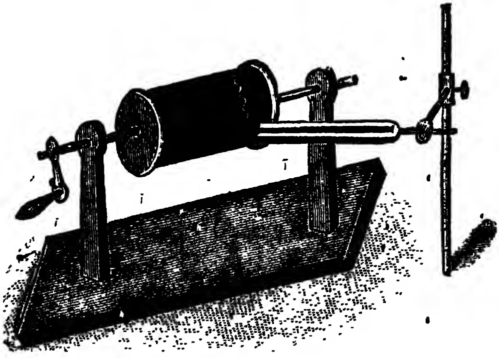


Fig. 86.

The pendulum of a (seconds) clock having a fine platinum end is caused to touch a drop of mercury, placed in a paraffin block, as it passes its lowest point, and is placed in the primary of an induction coil to make and break the current once in a second. Each time the circuit is broken a spark passes from the style to the cylinder and marks the paper on the wavy line, and the number of waves between two successive marks gives the frequency of the fork.

110. Electrical maintenance of vibration of a fork. An electromagnet is placed between the prongs of a tuning fork as shown in fig. 87. A current from one or two storage cells passes through the electromagnet and is interrupted by the vibrations of the fork itself. A platinum wire attached to one of the prongs just touches an adjustable screw held on the frame work and completes the circuit

when the prongs are at rest. But as soon as the circuit is completed the prongs are attracted inwards by the electromagnet and the current is broken. The elctromagnet now ceases to attract the prongs, which now being released spring outwards and set up the current. The current is thus automatically made and broken. The interruption of the current may also be affected by a platinum wire attached to one of the prongs dipping into a mercury cup, which would complete the circuit when the prongs are at rest

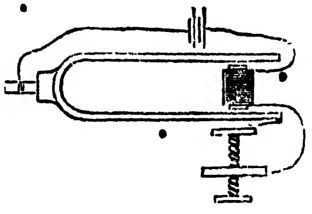


Fig. 87.

and would break the circuit when the prongs are attracted inwards. The electromagnet does work on the fork while the prongs are attracted, and the fork does work on the electromagnet while the prongs move outwards. If these quantities of work were equal, then on the whole no energy would be drawn from the battery and the vibrations would die away. But owing to (1) irregular contact and (2) self induction of the coils of the electromagnet, the work done by the battery is greater than the work done by the fork, and it is this extra work done by the battery that supplies the energy of sound radiated by the vibrating fork and also the energy lost by dissipation against friction.

111. Sonometer or Monochord. In Art 77 it has been shown that the frequency of the note given out by a string of length l and stretched by a tension T, while sounding its fundamental tone is given by

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$T = W \times g$; where W is the weight attached to the end of the string and g is the acceleration due to gravity.

m = mass of the string per unit length.

$= \pi r^2 \rho$ (r = radius of the string and ρ the density of its material.)

By measuring l, T and m the value of n is directly found out. The friction of the pulley changes to some extent the tension due to the weights when the sonometer is in the horizontal position. This difficulty is avoided by suspending the sonometer in a vertical position.

To find the frequency of a given note adjust the length of the vibrating string by the movable bridge until the sound given out by the string is in unison with the given note. It is not easy to effect exact unison with untrained ear, specially when the notes are of high pitch, since in such cases some of the harmonics of the fundamental may be mistaken for notes of the same pitch. The position of unison should, therefore, be adjusted both by increasing the length of the wire and decreasing it by the movable bridge, and when in unison, it should be examined that on increasing the length the note is perceptibly higher in pitch and on decreasing the length it is perceptibly lower.

Tuning effected by beats. Unless two notes are very loud and of very high pitch, beats are generally heard

when they are nearly of the same frequency. Vary the length of the string till beats are distinctly heard, specially when the sound is dying out, and then carefully adjust the position of the bridge till the beats vanish. This position should be found out by moving the bridge from either side of the correct point.

Tuning effected by resonance. If it is possible to hold the vibrating body on the board of the sonometer the string will be thrown into energetic vibrations by resonance, when it is in tune with the vibrating body.

If, therefore, it is found difficult to effect unison with the ear, then adjust the length of the string till the beats just vanish. Mount a light *paper rider* at the middle of the string and hold the vibrating body on the board. If the string be in tune with the vibrating body, then it will begin to vibrate energetically and will throw off the rider.

112 **Resonance**. Take a tall cylindrical glass jar nearly full of water and a tube of glass or some metal of about the same length and about one inch in diameter. Introduce the tube into the jar and support it vertically in a clamp. Hold the vibrating body near the open end of the tube and raise the tube gradually till there is *maximum resonance.* Measure the length of the tube above the water surface. Next raise the tube still further till a second position of resonance is found. In this case the sound is weaker in intensity than in the first case. As shown in Art. 66 when the tube sounds its fundamental

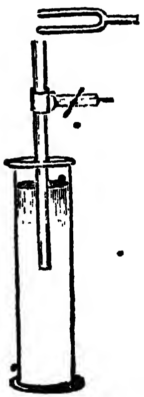


Fig. 88

tone, its length l is given by $l=\frac{\lambda}{4}$

When the tube sounds its first overtone, its length l' is given by $l'=\frac{3\lambda}{4}$.

As explained in Art. 64 the first loop is not formed exactly at the open end but a little outside. This distance is found by experiment and calculation to be about $\frac{1}{3}$ of the diameter of the tube. Correcting for this we get

$$l+\cdot 3d=\lambda/4 \ldots\ldots(\text{i})$$

$$l'+\cdot 3d=\frac{3\lambda}{4} \ldots\ldots(\text{ii})$$

where d is the diameter of the tube.

But $n\lambda=V=$velocity of sound in air inside the tube.

$$n=\frac{V}{\lambda}=\frac{V}{4(l+\cdot 3d)}=\frac{3V}{4(l'+\cdot 3d)}$$

We may eliminate $\cdot 3d$ however by taking the difference of (ii) and (i)

$$l'-l=\lambda/2$$

$$\therefore \quad n=\frac{V}{2(l'-l)}$$

The air column inside the tube may be taken to be saturated and at the temperature of water in the jar. V should therefore be taken to be the velocity of sound in saturated air at this temperature.

Experimental determination of the velocity of sound.

113. Velocity of sound in air. *Direct method.* Sound takes time to travel from one point to another. The

thunder is only heard an appreciable time after the lightning flash is seen; the flash of a gun is always seen a short time before the report is heard. The time which elapses between seeing the flash and hearing the report is the difference of the times taken by light and sound to travel from the source to the observer. But the velocity of light is about 186,000 miles per second, and therefore the time taken by light to travel over a few miles is infinitely small, and thus the interval between the instant the flash is seen and the instant the report is heard is practically the time taken by sound to travel from the source to the observer. To determine the velocity of sound directly, a gun is fired at a station and the report is heard a few miles apart at another station. By carefully measuring the time taken by sound to move from the first station to the second, and knowing the distance between the stations, velocity of sound in air is directly calculated.

Let d = distance between the stations,

t = time taken by sound to travel from one station to the other.

Then V = velocity of sound in air $= \frac{d}{t}$

But this result is subject to the following sources of error.

(i) Inaccurate measurement of t.

(ii) Temperature variation in the medium.

(iii) Hygrometric state of the air.

(iv) Wind.

(i) The observer does not perceive either the flash or the report exactly at the moment when the light and sound

reach him. The process of perception requires an appreciable time in each case, which is different for sight and hearing. Besides the observer does not record time exactly at the moment when he becomes aware of the flash and the report, and the interval between the instant the disturbance is perceived and the instant when it is recorded may be different in the case of light and sourd. Preception of sound again varies most probaby with the same observer, depending, among other factors, on the intensity of sound, loud sounds being perceived more quickly than faint ones

These difficulties may be avoided by dispensing with the observer altogether and recording time mechanically by some electrical arrangement. The instant of firing is recorded on the rotating drum of a chronograph by causing the bullet to cut a wire and thus break an electric current. At the receiving station the sound waves enter a wide cone closed at the further end by a diaphram. The waves strike the diaphram and thus drive it forward and break the electric current, the record being made on a rotating drum as before.

(ii) The temperature of air may not be the same all over between the two stations. The effect of rise in temperature is to increase the velocity given by

$$V_t = V_0\sqrt{1+\alpha t}$$

$$= V_0\left(1 + \frac{\cdot 00366}{2}\right) = V_0\ (1 + \cdot 00183)$$

The error due to this is reduced to a minimum by observing temperature at a number of points between the stations and taking the mean value.

It is to be noted in this connection that if air is heated in a closed vessel so that it can not expand, then its density remains the same but the elasticity increases. Air heated in this manner transmits sound more rapidly than at lower temperatures. If however air when heated is allowed to expand freely, its pressure and therefore the elasticity remain the same but the density becomes less and so it transmits sound more rapidly than when it is cooler.

(iii) Moisture. As previously considered aqueous vapour is lighter than air, and λ for aqueous vapour is 1·31 whereas that of air is 1·41.

As shown in Art 47.

Velocity in dry air at 760 and and $t°$ C

$$= V_{dt} = V_{mt}\sqrt{\frac{P - \cdot 378e}{P}}.$$

(iv) Sound is transmitted more effectively when it travels with the wind and is retarded when travelling in the opposite direction. This effect is eliminated by the method of reciprocal observations. Guns are fired at both stations simultaneously and the instant the sound reaches each station from the other is recorded. The mean of the times taken by sound to travel in opposite directions from one station to the other gives the correct time.

From the beginning of the eighteenth century the velocity of sound has been determined by the direct method by many observers. In 1708 Derham first noticed the effect of the wind. But he made no record of the temperature and the hygrometric state of the air, and the result of his experiment gave the velocity of air as

1142 ft/sec. In 1822 Arago and a number of French observers repeated the experiment at two stations at a distance of 18612·5 metres apart. The effect of the wind was eliminated by the method of reciprocal observations and the temperature was recorded at a number of points between the stations. They obtained for the velocity in dry air at $0°C$ $V_0 = 331{\cdot}1$ m/sec.

One of the most accurate measurements was made by Moll and Von Beach in 1823. Their value corrected for temperature and hygrometric state is 1092 7 ft./sec.

In the years 1862—66 Regnault devised the method of recording time electrically on the rotating drum of a chronograph, and performed a series of experiments and he gave, as the final result of his experiment, 330·6 m/sec. as the velocity of sound in air. In 1891 Mr. Stone of Cape Town Observatory made a series of very careful determinations and the final value obtained by him in dry air at $0°C$ was 1093 6 ft/sec.

The velocity of sound in dry air and at $0°C$ may thus be taken to be 1090 ft/sec or 332 m/sec.

$$V_t = V_0 \sqrt{1+\alpha t} = V_0 (1 + {\cdot}00183t)$$

This amounts to an increase of nearly 2ft. or 61 cm per degree centigrade rise in temperature.

114 Resonance method The experimental procedure is the same as in Art 112. The length between two consecutive nodes being found out and the frequency of vibration of the tuning fork being known, the velocity of sound is obtained from the relation given by $2nl = V$

Where l=is the length between two consecutive nodes,
n=frequency of the tuning fork,

V=velocity of sound in saturated air at the temperature of water in the vessel, the air column above the water surface inside the tube being assumed to be practically saturated. This result corrected for temperature and the hygrometric state, as explained in the previous article, will give the velocity of sound in dry air at 0°C. The correction for the diameter should be made if necessary as explained previously.

115 Laboratory method. Method of echoes. Take a metronome which beats loudly and adjust the length of its pendulum so that it vibrates slowly. Move it away from a reflecting surface, wall for example, until a position is found where the echo of one beat coincides with the sound of another beat heard directly.

Let d=distence of the wall from the metronome,

t=interval between two beats,

V=velocity of sound in air at the time of the experiment.

Then $t=\frac{2d}{V}$=time taken by the sound to move from the metronome to the wall and then back to the metronome.

$$V = \frac{2d}{}$$

116. Velocity of sound in water. This may be determined by the direct method as in the case of air. In 1826 Collodin and Sturm performed an experiment in the lake of Geneva. Two boats were moored at a distance of

about 8 miles apart The source was a large bell suspended from one boat inside water about one metre below the surface. The hammer striking the bell was worked by a lever so arranged that the moment it touched the bell it ignited some gunpowder. The receiver was a large horn-shaped trumpet about 3 metres long, the larger end of which was closed by an india rubber membrane and dipped into the water, and the other end was applied to the ear of the observer The observer recorded time the instant he heard the report. The value obtained was 1835 meters/sec. at 8·1°*C*

Threlfall and Adair made a series of careful measurements in sea water and found that the velocity increased to some extent with the intensity of the explosion. By using 9 oz guncotton the velocity was found to be 1742 meters/sec at 17·8°*C*, while by using 18 oz the value rose to 1942 meters/sec at 18·2°*C*.

117. Velocity of sound in solids. *Direct method.* The velocity of sound was determined by Biot and Martin, who took a cast iron tube about 951 meters long for this purpose. One end of the tube was struck with a hammer and an observer at the other end heard two sounds distinctly, the first transmitted by the metal and the second by the air. The interval between the two sounds was 2·5 seconds.

Let V_i = velocity of sound in cast iron

V_a = velocity of sound in air at the time of observation.

$$\text{Then } \frac{95100}{V_a} - \frac{95100}{V_i} = 2{\cdot}5$$

From which it is easily found that V_i is about ten times V_a.

The result obtained from Biot's experiment is not very accurate since the interval between the sounds was only 2·5 seconds and therefore too small to be determined accurately. Besides the tube was not a continuous mass of iron but formed a series of tubes joined together by lead.

117. Velocity of sound in solids and gases. Kund's dust tube experiment.

For this purpose a perfectly dry glass tube, about 5 ft. in length and 2 inches in diameter, is held horizontally in a framework. One end of the tube is closed by a moveable piston or tight fitting cork. A rod of glass or some metal of about the same length but 1/2 inch in

Fig. 89

diameter is fitted with a disc of cardboard or ebonite, just smaller in diameter than the tube; it is clamped firmly at its middle *B* horizontally, so that the disc end *C* projects into the tube some way up, and the axes of the tube and the rod are in line. A little dry lycopodium powder or corkdust is introduced inside the tube and strewn along its length in a fine line. On drawing a piece of cloth moistened with alcohol along the outer half of the rod, if it is made of glass, (resined leather if the rod is made of some metal), it is thrown into longitudinal stationary undulation, giving out its fundamental with a node at the middle and antinodes at the ends. The tube is now shifted laterally through a small extent, say 1 cm., at a time, until the dust inside the tube begins to move

when the rod squeaks. The position of the tube is now carefully adjusted so that exact resonance takes place and a clear loud note rings out. The dust inside the tube is blown at the antinodes and settles down at the nodes, where there is no motion. The velocity of sound in the rod being much greater than in air, many nodes may be formed in the tube, the first being at the closed end of the tube and the last near the disc. The disc transfers its energy of motion to the air in front of it and thus sets up longitudinal waves which advance through the tube and get reflected at its closed end. The direct and the reflected systems travelling in opposite directions set up stationary waves inside the tube. The longitudinal movements of the rod are small, but the amplitude of vibration in the air soon far exceeds that of the rod, and therefore the motion of the disc corresponds to that of a point much nearer a node than an antinode. The disc end therefore though an antinode for the rod, is practically a node for the air.

Let V_r = velocity of sound in the rod,

V_a = velocity of sound in air at the time of observation,

λ_r = wave length of the note given out by the rod in the rod,

λ_a = wave length of the same note in air,

n = frequency of vibration of the rod,

l = length of the rod,

d = length of the internode in the tube i.e. the distance between two consecutive nodes.

Then since $V_a = n\lambda_a$ and $V_r = n\lambda_r$

and $l = \frac{\lambda_r}{2}$, $d = \frac{\lambda_a}{2}$

Hence $\frac{V_r}{V_a}=\frac{2nl}{2nd}=\frac{l}{d}=\frac{\text{length of the rod}}{\text{length of the internode in the tube}}$.

If n be known then both V_a and V_r can be calculated from the above relations. The ratio of the velocities however is directly determined even when n is not known. By using rods of different materials and filling the tube with different gases, the velocities of sound in them are easily found out.

117. Temperature effect studied by Kund's tube. For this purpose the tube is surrounded by another wide glass tube and a current of steam is passed in the space between the two tubes. When the temperature becomes steady, the lengh of the internode in the tube is determined accurately.

We have $n\lambda_{100}=V_{100}$ and $n\lambda_t=V_t$

$$\therefore \quad \frac{V_{100}}{V_t}=\frac{\lambda_{100}}{\lambda_t}$$

$$=\frac{\text{length of the internode inside the tube at } 100^\circ C}{\text{length of the internode inside the tube at } t^\circ C}$$

where λ_{100} = wave length in air at $100^\circ C$
= twiee the length of the internode inside the tube at $100^\circ C$

λ_t = wave length in air at $t^\circ C$
= twice the length of the internode inside the tube at $t^\circ C$

V_{100} = velocity of sound in air at $100^\circ C$

V_t = velocity of sound in air at $t^\circ C$.

118. Ratio of the specific heats of a gas determined by Kund's tube.

We have $V=\sqrt{\frac{\gamma P}{\rho}}$

where V=velocity of sound in a gas at a certain temperature,

ρ=the density of the gas at pressure P at the temperature of observation.

V, P and ρ being known γ is found out from the above relation.

119. Velocity of sound in stretched strings. As shown in Art. 80 the velocity of propagation of a longitudinal wave along a stretched string is independent of the tension with which it is stretched and depends only on the the elasticity and density of its material.

$$V=\sqrt{\frac{E}{\rho}}$$

where E=the Young's modulus of the material of the string, and ρ=its density.

When both ends of the string are fixed, and it vibrates in its fundamental, there are two nodes at the fixed ends and a loop in the middle. In this case

l=the length of the vibrating string$=\lambda/2$

$\therefore\ V=n\lambda=2nl.$

Take a wire of steel or copper and stretch it over two bridges a few feet apart. The sonometer may be conveniently used for this purpose. Draw a resined leather along the string towards its free end, and find out the frequency of the note emitted by comparison in the usual way, and measure the length of the string. Then n and l being determined V is found out.

120. Determination of the Young's modulus of the material of a rod or wire.

Proceed as in Art. 119 and determine ρ, the density of the material of the string.

We have seen that $V = \sqrt{\dfrac{E}{\rho}}$ or $E = V^2\rho = 4n^2l^2\rho$

then n, l and ρ being known E is found out.

CHAPTER XII.

Musical Scale—Concord and Discord —Combination tones.

121. Musical intervals. A combination of two or more notes is called a *chord*. When the chord is agreeable it is called a *concord* or *consonance*; when disagreeable a *discord* or *dissonance*. Two or more notes though separately musical, may not produce a pleasant sensation when sounded together; they may thus form a discord. Again a series of discords following one another according to certain laws may produce pleasing effect, and thus lead up to a concord This distinction has been found to depend mainly on the relative frequencies or the ratio of the frequencies of the notes sounded. The ratio of the vibration frequency of a higher note to that of a lower note is called the *interval* between them. Thus if two notes have frequencies n_1 and n_2 and if $n_1 > n_2$, then the interval between them is denoted by n_1/n_2, and not by $n_1 - n_2$. The aim of music has been to arrange emperically notes of various pitch so that their combination may produce pleasing effect on our ear.

Thus it is found that a note C of frequency n so closely resembles a note c of frequency $2n$ that in music they are called by the same name. The interval between C and c is called an octave; C is an octave below c and c is an octave above C. Similarly notes c', c'' etc. having frequencies $2n$, $3n$ etc. are 2 octaves, 3 octaves above C and so on. The most concordant interval is $2:1$; next to this is the interval $3:2$;

and the next best is the interval 4 : 3. Again any three notes whose frequencies are as 4 : 5 : 6, when sounded together are concordant and form a *harmonic triad*; a harmonic triad together with an octave to the lower, constitutes a *major chord.* The distinctive character of a chord is not changed when any one of the notes is replaced by its octaves. Thus if four tones C, E, G and c form a chord, C, c, e, g form the same chord.

Any three notes whose frequencies are in the ratio of 10 : 12 : 15 when sounded together are slightly discordant but not altogether disagreeable. Three such notes with the octave to the lower constitute a *minor chord.*

122 Musical scale. A musical scale consists of a series of notes having certain relations to one another as regards their frequency of vibration. Between a note and its octave the human ear can distinguish a number of notes of definite frequencies. The *Gamut* or the *Diatonic* scale consists of eight notes called by names.

do	re	mi	fa	sol	la	si	do[1]

They are represented by the letters

C	D	E	F	G	A	B	C

and their vibration frequencies are proportional to the numbers

1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

or, in whole numbers, to

24	27	30	32	36	40	45	48

The intervals between successive notes are given by

$$\underbrace{1 \quad \tfrac{9}{8}}_{\frac{9}{8}} \quad \underbrace{\tfrac{9}{8} \quad \tfrac{5}{4}}_{\frac{10}{9}} \quad \underbrace{\tfrac{5}{4} \quad \tfrac{4}{3}}_{\frac{16}{15}} \quad \underbrace{\tfrac{4}{3} \quad \tfrac{3}{2}}_{\frac{9}{8}} \quad \underbrace{\tfrac{3}{2} \quad \tfrac{5}{3}}_{\frac{10}{9}} \quad \underbrace{\tfrac{5}{3} \quad \tfrac{15}{8}}_{\frac{9}{8}} \quad \underbrace{\tfrac{15}{8} \quad 2}_{\frac{16}{15}}$$

The scale is continued by taking the octaves of these notes *c*, *d*, *e*, *f*, *g*, *a*, *b*, *c* and then octaves of these last *c'*, *d'*, *e'*, *f'*, *g'*, *a'*, *b'*, *c'*, and so on.

do, the first note from which other notes of the scale are derived is called the *key-note* or the *tonic* of the scale. The key-note may have any frequency. The interval $\frac{9}{8}$ is called a *second*, $\frac{5}{4}$ a *third*, $\frac{4}{3}$ a *fourth*, $\frac{3}{2}$ a *fifth*, $\frac{5}{3}$ a *sixth* and so on.

The interval between do and mi is called a *major chord* and that between la and do a *minor chord*.

From the above it will be seen that there are three kinds of intervals $\frac{9}{8}$, $\frac{10}{9}$ and $\frac{16}{15}$. The interval $\frac{9}{8}$ is called a *major tone*, $\frac{10}{9}$ a *minor tone* and $\frac{16}{15}$ a *limma*. The difference between the *major tone* and the *minor* $\frac{9}{8} \div \frac{10}{9}$ or $\frac{81}{80}$ is called a *comma*, and the defference between the minor tone and the *limma* $\frac{10}{9} \div \frac{16}{15}$ or $\frac{25}{24}$ is called a *diesis*.

In addition to the notes already considered sometimes additional notes are used in music, which are obtained by raising or lowering the pitch of a note by a diesis or $\frac{25}{24}$. When a note is raised by a diesis it is said to be *sharpened*; and when lowered by the same interval it is said to be *flattened*.

123. Musical temperament. In music it is not possible always to use the same scale having only a definite set of notes, but it is often necessary to use scales having differrent key-notes. But it is practically impossible to arrange more than one key-note in instruments like the piano or the organ, in which the pitch of the various notes is fixed. Thus for example, if we want to construct a scale with *e'* as the key-note, then we at once find that the notes of the old scale do not fit into the new scale, because the interval between *e'* and *f'* in the old scale is $\frac{16}{15}$, whereas the interval between

the key-note and the next note above ought to be $\frac{9}{8}$. This difficulty can not be avoided even by using a more extended scale in which each note is sharpened and flattened, because the sharp of one note has not necessarily the same frequency as the flat of the note above. Thus in order that the same series of notes may be used for music written in different keys it is necessary to so alter the relative frequencies of the various notes that the notes belonging to the scale in any key will exactly fit into the scale with any other key. This process of adjusting notes is called *temperament* and the adjusted scale is called the *tempered scale*.

There are different methods of temperament. The one generally used is the system of *equal temperament*; so called because in this system the interval of the octave is kept unaltered and the errors are distributed equally over the remaining intervals. In this system the difference between the major and the minor tones is ignored and the limma is made just half of either. The interval between the key-note and its octave is divided into 5 tones and 2 semitones, a tone being $\frac{1}{6}$ of an octave and a semitone $\frac{1}{2}$ of a tone i.e. $\frac{1}{12}$ of the octave. The interval of a tone therefore is the sixth root of 2 and that of a semitone the 12th root of 2. Thus if the interval of the semitone or the tempered limma is represented by x, then

$$x^{12}=2$$

$$\text{or } x=1{\cdot}059.$$

The relative frequencies of the notes on the natural scale and the equally tempered scale are given below

	C	D	E	F	G	A	B	C
Natural scale								
	1·000	1·125	1·250	1·333	1·5000	1·667	1·875	2·000

Tempered scale

1·000	1·122	1·260	1·325	1·498	1·682	1·888	2·000

124 Concord and discord. We have seen in the previous article that when two notes are sounded together their combination is either *consonant* and is pleasing to our sensation, or is *dissonant* and is unpleasant to us. This distinction depends mainly on the ratio of the frequencies of the notes, that is, the interval between them ; it also depends on the absolute frequency and the harmonics present (when the sound is composite). It has also been seen in this connection that intervals, unison (1 : 1), octave (1 : 2), octave and the fifth (1 : 3), double octave (1 : 4), fifth (2 : 3) and fourth (3 : 4) are consonant.

The major third (4 : 5), the major sixth (3 : 5) together with the minor third (5 : 6) and the minor sixth (5 : 8) are less consonant.

The second and the seventh, both major and minor are dissonant.

According to Helmholtz discord is always due to beats, and the unpleasantness of a given combination depends partly on the strength of the beats and partly on their frequency. Thus if we start with two simple tones in unison, and gradually increase the frequency of one of them the number of beats produced per second also increases gradually. Very slow beats are not unpleasant, but with the rapidity of the beats the jarring or the rattling effect increases till for a certain frequency of the beats the discord is a maximum. If the frequency of the beats is still further increased the unpleasantness gradually decreases and ultimately vanishes.

These limits are different for sounds of different pitch. For sounds of medium pitch. lying within the range of the voice, beats at the rate of 2 to 10 per second are perceived separately and the effect is not unpleasant. Above 10 the beats are not perceived separately and begin to become unpleasant. The unpleasantness becomes maximum at 30, above which the effect becomes less and less unpleasant, and ultimately above 70 it vanishes. The sensation produced by beats is similar to the optical sensation produced by the flickering of a gas flame. With the number of flickers per second the unpleasantness at first increases and attains a maximum value, and then it decreases and finally vanishes when the flickers are so rapid as to produce the effect of a continuous impression. The frequency corresponding to the maximum unpleasantness is smaller for the eye than for the ear owing to greater persistence of visual impression.

It is also found that two notes when sounded together are dissonant even when the interval between them is far beyond the limiting interval considered above. Thus if two notes instead of initially being in unison, one is the octave of the other, we still hear beats when they are slightly out of tune. If the frequency of one is 250 and that of the other 512, we hear 12 beats per second. The reason underlying this phenomenon is that the lower note is not a pure tone, but contains some harmonics, the most prominent of which has frequency 500, and this harmonic beats with the fundamental tone of the other, the number of beats per second being 12, that is the difference of 512 and 500.

It may be assumed that a simple tone when it reaches the fluid in the *Cochlea*, excites a select group of fibres;

the middle members of the group having exact agreement are excited strongly and the extreme members whose periods nearly agree with that of the tone are very feebly excited. When two simple tones of nearly the same frequency reach the ear simultaneously, the two groups of fibres that are excited overlap, in other words, some fibres are common to both, and their vibrations being the resultant of two *S.H.Ms.* of nearly the same period, varies in amplitude from the sum to the difference of the amplitudes of the two components, causing a periodic waxing and waning in the intensity of the sound which are perceived as beats. When the beats are sufficiently slow they are perceived separately as separate maxima of sound. If, however, the beats are so rapid that the auditory nerve cannot recover completely between the two stimuli the effect is a discord. Again when the beats are very rapid so that none of the fibres are excited by both tones then the unpleasantness vanishes.

If it is further assumed that each fibre responds to a tone and also its harmonics, it will explain why the interval of the octave is so consonant, and also why two notes, when the frequency of one is nearly the same as the octave of the other, produce beats with unpleasant effect.

125. **Combination or Resultant tones.** We have so far confined our attention to vibrations of very small amplitude in which the restoring force is proportional to the displacement. In such cases the law of superposition holds good and the phenomena of interference are produced.

When however the amplitudes are large the restoring force is not proportional to the displacement and the law of superposition ceases to hold good. For example when an unsymmetrical membrane like the drumskin of the ear,

or a small air cavity is subjected to large vibrations, the restoring force is not proportional to the displacement. In such cases secondary waves are set up which appeal to the ear as *combination tones*, also called *resultant tones*

When the vibrations set up in the air by a sounding body are so energetic as to exceed the limit of the law of superposition, secondary waves are set up, which correspond to the harmonic tones of the sounding body and are called *self-combination tones.*

Again when two notes are so intense that the law of superposition no longer holds good, they set up secondary waves, which combine and give rise to combination tones whose frequencies are either the sum or the difference of the frequencies of the primaries. Those whose frequencies are the difference of the frequencies of the primaries are called the *difference tones*, and the others whose frequencies are the sum of the frequencies of the primaries are called the *summation tones.*

Thus for exmple if m and n be the frequencies of two simple tones, then the frequency of the difference tone is $m-n$. This is called the first difference tone since it can itself form difference tones of the second order and so on. Similarly $m+n$ is the first summation tone which can itself form second summation tones and so on. Thus the two simple tones may give rise to a series of combination tones, (m,n) primaries, $(2m,2n)$ self combination tones, $(m-n)$ first difference tone, $(m+n)$ first summation tone, $(2m-n)$ summation tone of the first difference with one primary and so on.

Combination tones were first discovered by Sorge, a German organist in 1745. They were afterwards indepen-

dently discovered by Tartini, an Italian violinist, in 1754. It was Helmholtz however who studied the subject fully.

To render the combination tones audible the primaries must be loud and should be sustained. They are conveniently heard when two organ pipes are blown hard; when a double whistle is blown hard; or two gongs are struck near together close to the observer. Helmholtz obtained them readily with the siren.

These tones are not generally reinforced by resonators, showing that they are produced in the ear and have no existence in the outside air, for if they existed in the air, they would have been reinforced by suitable resonators held near the ear.

In certain cases combination tones may however be formed in the air. For example in the case of a harmonium or a siren the sound waves just on their production escape from or into a closed air space, where the vibrations are so energetic that the restoring force is not proportional to the displacement, and thus secondary waves are set up producing combination tones.

Thomas Young tried to explain the production of combination tones as being due to a succession of rapid beats following one another like the impulses of an ordinary musical note. At his time summation tones were unknown and his theory gave a ready explanation of the difference tones, whose frequencies, like those of beats, are equal to the difference of the frequencies of the primaries. Helmholtz afterwards proved mathematically that when two primary tones agitate energetically the same body or the same portion of air, their combined effect would produce two derived systems whose frequencies are respectively equal to the sum and the

difference of the frequencies of the primaries. He next brought his result to the test of experiment and succeded in demonstrating the existence of summation tones. Young's theory fails to explain the formation of the summation tones. Besides if combination tones were due to beats, they could be heard even when the primaries were feeble, because when two tones of the same intensity produce beats, the intensity of the beats changes alternately from zero to four times the intensity of either of the primaries. But combination tones are not heard with feeble primaries. That the combination tones are not due to coalescence of rapid beats is also evident from the fact that under favourable conditions both beats and combination tones are heard together.

The subject of combination tones is very complicated and a complete investigation of it is not possible at this stage. We may however form some idea of the mode of production of these tones from the following instructive analytical investigation of Poynting and Thomson.

Let two reeds of frequencies n_1 and n_2 mounted on the same wind-box, be blown by air whose average excess of pressure is P. Then assuming that P remains constant and the issuing disturbance is proportional to the pressure excess and the amplitude a of the reed, the issuing disturbance may be represented as $Pa \sin 2\pi n_1 t$ But the vibrations of the reed changes P with a periodicity, so that the pressure excess may be represented at any rate by

$$P\{1 + b \sin (2\pi n_1 t + e_1)\}$$

The issuing disturbance may thus be put as

$$P \sin 2\pi n_1 t\{1 + b \sin (2\pi n_1 t + e_1)\}$$

$$= Pa \sin 2\pi n_1 t - \frac{Pab}{2}\cos (4\pi n_1 t + e_1) + \frac{Pab}{2}\cos e_1 \ldots (i)$$

In (i) both the fundamental and the octave are present By correcting the pressure for the octave by introducing a term the next harmonic is obtained and so on.

The pressure excess as modified by the second reed may similarly be represented by $P\{1 + c \sin(2\pi n_2 t + e_2)\}$

Now if both reeds are sounded together and the variation in the pressure excess due to the first reed is neglected, then the issuing disturbance is represented by

$$Pa \sin 2\pi n_1 t\{1 + c \sin(2\pi n_2 t + e_2)\}$$

$$= Pa \sin 2\pi n_1 t + \frac{Pac}{2}\cos\{2\pi(n_1 - n_2)t + e_2\}$$

$$- \frac{Pac}{2}\cos\{2\pi(n_1 + n_2)t + e_2\}\ldots\ldots(\text{ii})$$

In (ii) the first difference tone and the first summation tone are present. By correcting the pressure for these tones, the next higher tones are obtained, and so on.

We thus see that when several instruments are played together in the music of a concert, each instrument gives out not only its fundamental tone, but also some of its overtones and self-combination tones. Then there are resultant tones, both difference tones and summation tones. All these move through the same air and interfere with one another. The physical condition of the air through which the waves travel is too complicated to be realised even in imagination.

B. Sc. QUESTIONS

CALCUTTA UNIVERSITY

1909.

1. What is meant by the interference of sound waves? Describe an experiment showing the interference of sound waves, and explain how it may be used to measure the wave length of the note employed. Art 98.

2. Give the laws governing the transverse vibration of strings. The density of steel wire being 7·7 and that of copper wire 8·9 what must be the ratio of the diameters of two wires, one steel and the other copper, of equal length, so that when stretched by the same force they may give the same note? Art. 77.

$$n_1 = \frac{1}{2l_1 r_1}\sqrt{\frac{T_1}{\pi\rho_1}}; \quad n_2 = \frac{1}{2l_2 r_2}\sqrt{\frac{T_2}{\pi\rho_2}}$$

But $l_1 = l_2$, $T_1 = T_2$, $n_1 = n_2$

$$\therefore \frac{r_2\sqrt{8\cdot9}}{r_1\sqrt{7\cdot7}} = 1 \quad \text{Or} \quad \frac{r_1}{r_2} = \frac{\sqrt{8\cdot9}}{\sqrt{7\cdot7}}$$

3. Describe the construction and operation of a gramophone. Art. 91.

4. On the wave theory of light, prove (*a*) the connexion between the velocities of light in two media and the index of refraction, and (*b*) the equality of the angles of incidence and reflexion in the case of light failing on a reflecting surface. Art. 62.

1910

1. Show that a simple harmonic motion in a straight line may be resolved into two circular motions. Art. 90

2. Define Young's modulus of Elasticity. Taking the modulus of steel wire as 1800×10^6 grammes per sq. cm, find the elongation of such a wire 10 meters long and 1 sq. mm. in cross sectional area when stretched by a weight of 10 kilos. Art. 16

$$E = \frac{FL}{sl} \text{ units of force per unit surface.}$$

$$= 1800 \times 10^6 \times 981 \text{ dynes per unit surface.}$$

$L = 10 \times 100$ cm.

$F = 10 \times 1000 \times 981$ dynes. $l = ?$

$s = \cdot 01$ sq. cm.

3. What is meant by resonance? (Art. 68.) In what different ways can the sound emmited by a vibrating turning fork be made audible at a considerable distance? (Arts 68, 72, 86, 51, 54 and 56) Show that the different means of producing this effect are not strictly analogous to one another.* How can the vibrations of the prongs of a tuning fork be made visible at a distance and how can they be utilised to measure small intervals of time? (Arts. 34 and 109)

* *Ans.* In the case of resonance the sound emitted by the fork is greatly intensified only when the tube has definite length or the resonator has a definite size, so that the natural period of the volume of air inside the tube or the resonator is the same as that of the fork. In the case of the sounding board, however, the vibrations of the fork are communicated to the entire mass of the box and the air within it. The vibrations of the box and air within it are *forced*, their natural periods of vibration being generally different from that of the fork. In the case of resonance a tube responds to those notes only which it is fitted to yield, but the same sounding box is used for tuning forks of different frequencies. In the case of conjugate mirrors, speaking tubes or reflecting gallery the intensity is not increased as in the case of a sounding box, but the sound is heard at a considerable distance without much loss in intensity.

The arrangement of the fork and the drum (Art 109) may be used as a chronograph to measure small intervals of time. An arrangement is made for breaking the current in the primary circuit at the beginning and end of a given interval of time. The interruptions of the current produce sparks which mark the paper on the wavy line. If n be the frequency of the fork it will trace n waves in one second. If the number of waves between two marks produced by the interruptions of the cirrent be m, then the interval between the interruptions is given by m/n sec.

4. How can the existence of nodes and antinodes in a sounding organ pipe be demonstrated? Art. 69

5. Describe that experiment of Melde's, which has for its purpose the illustration of the laws governing the vibration of strings. Art 79.

The tuning fork makes 128 vibrations per second; a mass of kilogramme is placed in the scale pan, the wire is of platinum of sp. gr. 21, the diameter is ·5 mm. What must be the effective length of the wire for a node to be formed at the middle?

Ans. When the string vibrates in unison with the fork

$$n=\frac{1}{2l}\sqrt{\frac{T}{m}}$$

n = the frequency of the fork = 128,

T = tension with which the string is stretched = 1000×981 dynes.

m = mass of the wire per unit length = $\pi \times {\cdot}25^2 \times 21$.

l = distance between two consecutive nodes = half the length of the string.

6. A man standing on a train which moves at the rate of 108 kilometers per hour blows a whistle the pitch of which corresponds to 1200, vibrations per second. What is the apparent pitch of the whistle to a person standing at a point in the direction towards which the train moves? Temperature of the atmosphere at the time of observation was $30^0 C$; velocity of sound in air at $0^0 C$ = 33060 cm/sec. Art. 100,

$n'=n\frac{V+w-v_0}{V+w-v_s}$; in this case $w=0$ and $v_0=0$

$$n'=n\frac{V}{V-v_s}$$

n'=required pitch (?)
n=pitch of the whistle= 1200
V=velocity of sound in air at the time of observation

$$=V_0\left(1+\frac{\alpha t}{2}\right)=33060\left(1+\frac{\cdot 0036\times 30}{2}\right)\text{cm/sec.}$$

$$v_s=\text{velocity of the source}=\frac{108{,}00{,}000}{60\times 60}\text{cm/sec.}$$

1911

1. What do you understand by simple harmonic motion (Art 19) Explain carefully how could you represent it graphically. (Art 27) Show how to combine two harmonic motions of the same period and equal amplitudes.

1. when they are in the same direction.

2. when they are at right angles to each other. The difference of phase in both cases being 90^0. (Arts. 31 and 32).

2. What is the pitch in the case of sound? How is it accurately determined? (Arts. 11 and 190).

3. In what essential point do transverse vibrations of a rod differ from the transverse vibrations of a string? Comparing a vibrating tuning fork with a vibrating rod supported in a certain way, show by a diagram that the ends of a tuning fork alternately approach and recede from each other, and that the attachment of of the stem does not interfere with the vibrations. (Arts 89* and 92)

* *Ans.* The rod being clamped at the middle there will be a node at the clamped point and two antinodes at the free ends.

4. Indicate the different ways in which the vibrations of air column of an organ pipe may be started and obtained. What effect is produced on the sound emitted by an open organ pipe (*a*)

by partially (*b*) by completely closing the open end of the pipe? Arts. 68 and 69.

1912

1. What are the characteristics of Simple Harmonic Motion? Arts. 19 and 27.

Trace, on the squared paper supplied, a complete closed curve to show the result of combining two simple harmonic motions at right angles to each other, that start from the same point with the same phase and have frequencies in the ratio 2 : 3. Art. 33

2. Define Young's modulus of elasticity. What will be the elongation produced in a steel wire one metre long and of one square millimetre cross section, when it is streched by a weight of a Kilogram, Young's modulus for steel being 2×10^{12} dynes per square centimetre. Art. 16. and Ques. 3, 1910.

3. Carefully explain what goes on in a sounding body and the atmosphere in its neighbourhood. What do you understand by the wave length of sound? Arts. 9 and 39.

4. Describe the distribution of nodes and loops that are formed in an organ pipe and in a string fixed at both ends. How is their existence made manifest? Tabulate the series of overtones present in the various cases. Arts. 68 and 69.

5. What are beats? How do they arise? Two tuning forks of frequencies 10 and 12 are sounded together. Show graphically what will be the number of beats. Under what conditions do beats produce resonance? Arts 99 and 70.

1913

1. A particle moves with uniform speed in a circle. Show that the motion may be resolved into two simple harmonic motions at right angles to each other. How do they differ in phase and amplitude? (Art 30).

Show how the potential and kinetic energies of a particle executing simple harmonic motion vary. Art. 20.

2 How has it been concluded that a sounding body e g. (*a*) a tuning fork, (b) the air column in an organ pipe, is in vibratory motion ? (Arts. 34, 109 and 69). To what physical characteristics do the loudness and the pitch of a musical note correspond ? Art. 11.

3. Describe the distribution of nodes and loops formed (*a*) in a closed organ pipe, (*b*) in a thin steel rod fixed at one end. How are their lengths to be adjusted so that they may sound in unison ? The velocity of sound in air is 33740 cm/sec and that in steel 5.2×10^5 cm /sec. Arts. 68 and 89.

4. Describe any two methods of determing the velocity of sound in air. Explain the various steps of the process in each case. (Arts. 113 and 114). How does the velocity of sound depend on the temperature of the medium ? Art 47.

1914

1. Define the terms amplitude, period and phase as applied to simple harmonic motion. (Art 25). Find the resultant of two simple harmonic motions, at right angles to one another, which have the same period and amplitude, their phase difference being 90^0 (Art 32). Indicate a method of experimentally demonstrating the accuracy of the result obtained. Art. 34.

2. Describe the distribution of nodes and loops (*a*) in an open organ pipe, (*b*) in a thin rod, fixed at one end and vibrating longitudinally (Arts 68, 89 and 69). How is their existence demonstrated ? describe a method by which the velocities of sound in various gases may be compared. Art. 117.

3. Define wave length and pitch of a musical note. How are they related ? (Arts. 10 and 39). Explain a method of determining each (Arts. 67 and 97).

When two notes of nearly equal pitch are sounded together, what effect is produced in the ear and in the air outside ? Arts. 88 and 99...

4. Find the velocity of sound in air (at any moment), from the following data :—

Pressure of the atmosphere = that of 760 mm of mercury.

Density of air = ·001293.

Ratio of the specific heat = 1·41

Temperature of the air = $10°C$. Arts. 46 and 47.

sound is emitted by a source placed at one end of a long iron tube and two sounds are heard at the other end at an interval of 2·5 seconds. If the length of the tube is 951·25 meters, find the velocity of iron.* Art 117.

1915

1. Define the following :—period, amplitude and phase. (Art 25). Show that the motion of the bob of a simple pendulum is appoximately simple harmonic. A particle moving with a simple harmonic motion *has a period of ·001 sec. and amplitude ·5 cm. Find its acceleration when it is ·2 cm from its mean position and its maximum velocity.

Ans. Acceleration $= -\left(\frac{2\pi}{T}\right)^2 x$ Art 24.

$$= -\left(\frac{2\pi}{\cdot 001}\right)^2 \cdot 2 \text{ cm/sec}^2$$

Velocity $= wa \cos wt$ (Art 26)

But $\cos wt = \sqrt{(a^2 - x^2)}/a$ where a is the amplitude and x the displacement.

$\therefore$ Velocity $= w\sqrt{a^2 - x^2}$, and this is maximum when $x = 0$.

The maximum velocity $= wa = \frac{2\pi}{T}\, a = \frac{2\pi}{\cdot 001} \times \cdot 5$cm/sec.

* **Simple pendulum.** It consists of a heavy particle suspended by a perfectly flexible weightless string from a fixed point, about which it swings in a vertical plane without any friction. Such a pendulum is an ideal one, but we may approximate to it by suspending a small metal sphere by a long and very thin thread.

In fig. 90 OP represents a simple pendulum in its position of rest. Let m be the mass of the pendulum and l its length. When the bob of the pendulum is drawn aside to Q, the forces acting on it are (1) its weight mg acting vertically downwards, and (2) the tension along its length QO. The force acting on the bob tending to bring it to its position of rest acts tengentially along QT. Since the tension has got no component along the tangent, the resultant force acting on the bob along $QT = mg \sin \theta$.

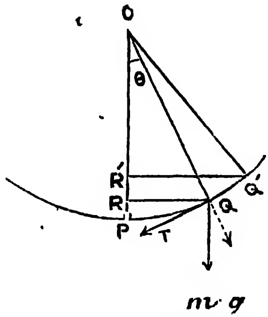


Fig 90.

When θ is very small $\sin \theta = \theta - \frac{\text{arc} PQ}{}$

$\therefore$ the acceleration with which the bob is moving $= \frac{g \text{ arc } PQ}{l}$

$= \mu$ arc PQ

Where μ is a constant and equal to $\frac{g}{l}$, i. e. acceleration of the bob is proportional to its displacement from the mean position. It therefore executes a $S.H.M.$

T=time period $= \frac{2\pi}{\sqrt{\mu}} = 2\pi \sqrt{\frac{l}{g}}$ Art. 24.

Another proof. Let the bob of the pendulum move from Q' to Q in time t.

RR'=the vertical height through which the bob has fallen in time t.

$l = (\cos \theta - \cos \phi)$; $\phi = \angle Q'OP$.

$\therefore$ Work done in the fall $= mg\, RR' = mgl\,(\cos \theta - \cos \phi)$

$=$ kinetic energy of the bob $= \frac{1}{2} m$ velocity^2.

Velocity of the bob $= l w$; $w =$ angular velocity $= \frac{d\theta}{dt}$

$\therefore$ acceleration with which the bob is moving $= l\frac{dw}{dt} = \frac{d^2\theta}{dt^2}$

Hence $\frac{ml^2w^2}{2} = mgl(\cos\theta - \cos\phi)$;

or $\frac{lw^2}{2} = g(\cos\theta - \cos\phi)$

Differentiating with respect to t we get

$l.w.\ \frac{dw}{dt} = -g\sin\theta\frac{d\theta}{dt}$

or $l\frac{d^2\theta}{dt^2} = -g\sin\theta$.

When θ is very small $\sin\theta = \theta = \frac{\text{arc}}{l}$

$\therefore \frac{ld^2\theta}{dt^2}$ = acceleration $= -\frac{g}{l}$ arc. $= \mu$arc. Where μ = constant $= -\frac{g}{l}$.

i. e. the acceleration is proportional to the displacement. The motion of the bob is therefore simple harmonic.

2. Describe a method by which the velocity of sound in a gas may be determined and compared with those in other gases. Art. 117.

The velocity of sound in air at $0^{\circ}C$ is 332 meters per second. Find the shortest length of a tube open at both ends, that will be thrown into rosonant vibration by a fork whose frequency is 250, when the temperature of the air $51^{\circ}C$.

$V_{51} = 3320^{\circ}\ (1 + \cdot 0018 \times 51) = n\lambda = 2nl$

$n = 256$

$l = ?$

3. A litre of hydrogen at normal temperature and pressure weighs ·0896 gramme. Find the velocity of sound in hydrogen at a temperature of $16^{\circ}C$, when the pressure is 750 mm, the ratio of the specific heats (hydrogen) being 1·4 [Density of mercury 13·6 and $g = 980$].

$v = \sqrt{\frac{\gamma P}{\rho}}$; $P = 75 \times 13{\cdot}6 \times 980$ dynes

$$\rho = \frac{\cdot 0696}{10,000}$$

$$\gamma = 1\cdot 4$$

$$V_{16} = V_0(1 + \cdot 0018 \times 16)\ \text{cm/sec.}$$

4. On what do the loudness, the pitch, and quality of a musical sound depend ? Arts 11 and 84

Explain briefly how these may be graphically represented. How would you propose to determine the pitch of a note experimentally ? (Figs. 8 and 9 Arts. 107 and 117.)

5. Deduce the laws of reflection and refraction of light on the wave theory. Art 63

1916

1. Define amplitude, phase and period of a simple harmonic motion. (Art 26). Find the period, given that the magnitude of the force at a distance x is μx (Art 24). A pendulum which beats once in two seconds at A gains two beats in an hour at B ; compare the weights of the same substance at the two places.

T_a = time period at $A = 2$ sec.

$= 2\pi\sqrt{\dfrac{l}{g_a}}$; g_a = acceleration due to gravity at A

T_b = time period at $B = \dfrac{60 \times 60}{(30 \times 60) + 2}$ sec.

$= 2\pi\sqrt{\dfrac{l}{g_b}}$; g_b = acceleration due to gravity at B

$$\frac{\text{Weight of a substance at } A}{\text{Weight of the same substance at } B} = \frac{\text{mass of the substance} \times g_a}{\text{mass of the substance} \times g_b} = \frac{g_a}{g_b} = ?$$

2. Indicate some method of experimentally determining the velocity of sound in solids. (Arts. 116 and 117). Young's modulus for steel is 210×10^{10}, and the specific gravity is 7·8. Find the velocity of sound through a steel bar. How would the temperature of the bar affect the result ? Art 117.

Increase of temperature decreases the elasticity of the steel rod.

$$V=\sqrt{\frac{E}{\rho}};\ E=210\times10^{10}\text{ dynes. }\ \rho=7\cdot8\ ;\ V=?$$

3. Explain the following terms :—pitch, beats, overtones. Show, by means of diagrams, the relations between these quantities. Arts 83, 76 and 99.

4. Describe and explain with suitable diagrams, the nature of the motion which constitutes sound. Art. 9

An open organ pipe emits its fundamental note. Find its length if the velocity of sound in air=33000 cm/sec, and it vibrates in unison with a violin string of length 33 cm, under a stretching force of $7\cdot89\times10^{6}$ dynes, the mass of the string per cm, being ·0042 gramme.

$$n=\text{frequency of the violin string}=\frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$=\frac{1}{2\times33}\sqrt{\frac{7\cdot89\times10^{6}}{0\cdot42}}$$

In the open tube $2nl'=V=33000$; where l'=length of the string sounding its fundamental, and l'=length of the pipe.

$$\frac{2l'}{2\times33}\sqrt{\frac{7\cdot89\times10^{6}}{\cdot0043}}=33000\quad l'=$$

1917.

1. Define simple harmonic motion, period, phase and amplitude. (Arts 19, and 25). Find the resultant of two simple harmonic motions of the same period but differing in phase and amplitude. Deduce the results in those cases where the phases differ by o, $\pi/2$, π and $3\pi/2$ respectively and the amplitudes are (*a*) equal, and (*b*) unqual. Illustrate your answer with neat pencil sketches on a squared paper. Arts. 31 and 32

2. State how Newton tried to explain the discrepancy between the velociiy of sound as determined by experiment and as deter-

mined from his expression $V = \sqrt{}$ What is Laplace's correction? Determine the velocity of sound at $N. T. P.$ and deduce the change in velocity per centigrade degree rise in temperature if $\alpha = \cdot 00367$. Arts 46 and 47.

3. Give a brief account of Kund's experiment. If the length of the rod is 1 metre, the density of the material 8 grams per c. c. and its Young's modulus $7 \cdot 2 \times 10^8$ grams per sq cm. find the distance between the heaps in the tube when it is filled with CO_2 at $25^\circ C$. Velocity of sound in CO_2 at $t^\circ C = (260 + \cdot 478t)$ meters per sec.

$$\text{Velocity of sound in the rod} = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{7 \cdot 2 \times 10^8 \times g}{8}} \text{ cm/sec.}$$

$$\text{Velocity of sound in } CO_2 \text{ at } 25^\circ = (260 + .478 \times 25) \times 100 \text{ cm/sec.}$$

$$\frac{\text{Velocity of sound in the rod}}{\text{Velocity of sound in } CO_2 \text{ at } 25^\circ C}$$

$$= \frac{\text{length of the rod}}{\text{distance between consecutive heaps in the tube}}$$

$$= \frac{100}{\text{distance between consecutive heaps in the tube.}}$$

Distance between consecutive Heaps=?

4. Describe accurately how you can determine the velocity of sound along a stretched wire in the laboratory. Art. 119

5. State the fundamental assumptions made by Huyghens, with regard to the wave theory of light. Explain the phenomenon of refraction and deduce Snell's law with the help of the theory.

Arts. 58, 63.

1918

1. Define the terms wave length, amplitude, and frequency (Arts. 25 and 39) Assuring that the velocity of water wave of wave length λ is given by the formula $v^2 = \frac{g\lambda}{2\pi}$ calculate, the velocity

and frequency of a wave when the distance from one crest to the next is observed to be 55 ft* What are stationary waves, and what is the difference between them and ordinary waves? Art. 66.

. Illustrate your answer with neat pencil sketches on a squared paper. Define the terms nodes and loops, and explain how are they formed. Art. 66.

* *Ans.* $v^2 = \frac{g\lambda}{2\pi}$ But $\lambda = 2 \times 55$ ft.

$\therefore \quad v^2 = \frac{g}{2\pi} \times 2 \times 55 = \frac{55g}{\pi}$

or $\quad v = \sqrt{\frac{55 \times g}{\pi}}$ ft/sec.

$n\lambda = v \quad \therefore n = \sqrt{\frac{55 \times g}{\pi}} \div 2 \times 55$

2. Obtain an expression for the velocity of sound in air, in terms of the pressure and the density. (Art. 44). Calculate the velocity of sound in air at $30^0 C.$ when the barometric height (corrected) is 755 mm. $\alpha = \cdot 00367$; $\gamma = 1 \cdot 4$ (Art. 47).

3. Distinguish clearly between stationary waves and progressive waves. Explain graphically or mathematically, the formation of nodes and antinodes when two trains of exactly similar waves travel along the same line in opposite directions. Art. 66.

4. Explain Doppler's principle, and describe how it can be easily demonstrated. Art. 100.

Two trains are approaching from opposite directions with the same speed of 100 ft./sec. The whistle of the first train is of frequency 1024. Find the variation of the apparent pitch calculated by an observer in the second train as the trains pass, supposing there is no wind and that the velocity of sound in air is 1116 ft/ sec.

In this case $n' = n\frac{V + v_0}{V - v_s}$ since $w = 0$ and the motions of the source and observer are in opposite directions.

$n = 1024$
$V = 1100$ ft/sec. $n' = ?$
$v_0 = v_s = 100$ ft/sec.

5. Give a general defination of the term Elasticity and prove that, in the case of a gas, the elasticity at constant temperature is numerically equal to the pressure. Calculate in C. G. S. units the value of Young's modulus, in the case of a wire, from the following data :—Arts, 12 and 16

Mean extension for 6 kgm.	$=0 \cdot 537$ mm.
Radius of wire	$=0 \cdot 675$ mm.
Length of wire	$=250$ cm

1919

1. Find the velocity and acceleration of a point executing simple harmonic motion. (Arts. 22, 26) Show that the motion of a simple pendulum oscillating into a small amplitude is simple harmonic in character. (Page 184). A point describes simple harmonic vibrations in a line 4 cm. long. Its velocity when passing through the centre of the line is 12 cm/sec. Find the period.

Velocity when passing through the central position $= wa = 12$

But $a = 4$ $\qquad w = \dfrac{12}{4} = 3 = \dfrac{2\pi}{T}$

$$T = \frac{2\pi}{3} \text{ sec. [Page 183]}$$

2. Describe Melde's experiment on vibration of a string. A fork vibrates along the length of a string that is attached to one prong and is stretched by a load $41 \cdot 8$ grammes. If the length of the string be 20 cm. and its mass per unit length be $\cdot 025$ gramme, and if the string vibrates in 4 segments, calculate the frequency of the fork. Art. 79.

In this case frequency of the fork is double the frequency of the string $= 2n$.

n = frequency of the string $= \frac{1}{2l}\sqrt{\frac{T}{m}}$;

$T = 41 \cdot 8 \times g$ dynes
$m = \cdot 025$ gram.
$n = ?$

l = length of the string between two consecutive nodes $= 20/4$ cm.

3. Explain by drawing suitable graphs, the formation of beats when two notes of nearly the same pitch are produced. Prove that the frequency of the beats is equal to the difference of the frequencies of the given notes. What is the effect of the beats on the concordance of musical notes? Arts. 99 and 124.

4. Prove that $V = \sqrt{\frac{E}{\rho}}$; where V is the velocity of propogation of longitudinal waves in a solid of Young's modulus E and density ρ. (Art. 44)

Explain briefly how by Kund's apparatus this velocity can be experimentally determined. Art. 117.

1920

1. Describe an arrangement for the production of Lissajou's figures. (Art. 34) State how they may be projected on a screen. Draw on a squared paper the curves traced by a point impressed with two S. *H. Ms* at right angles to one another which start so that the extreme positive elongations occur simultaneously when (*a*) their amplitudes are equal and periods are as 2 : 3; and (*b*) when the amplitude and period of one are double those of the other. Also draw the curve in the latter case when the slower vibration is impressed on the point as it passes the central position. Draw neat pencil sketches to show how the figures will be affected if the frequencies are not exactly but very nearly as 2 : 1 (Arts. 32 and 35).

2. Explain briefly the resonance column method for finding the velocity of sound in air. (Art. 112) The velocity of sound in air at 30° C and saturated with aqueous vapour is found to be

350 meters per second. Calculate the velocity in dry air at *N. T. P.*

Mass of 1 c.c. of air at *N.T.P.* = ·001293 gramme.

Density of acqueous vapour = ·62

Barometric height (corrected) = 760 mm.

Maximum vapour pressure at 30°*C* = 31·5 mm, Art. 47.

3. Describe any method for finding experimentally the velocity of propagation of transverse waves along a stretched string. Does the velocity depend on the tension? Indicate the variation of the velocity with the tension, if there be any, by a graph.

Ans. By the sonometer $v=\sqrt{\frac{T}{m}}$ Art. 48

or v^2 = a constant $\times T$

Comparing this with the general equation of a parabola $y^2 = 4ax$ (where a is a constant), we find that this relation of the velocity with the tension plotted graphically will give a parabola.

4. Explain Doppler's principle and how it can be demonstrated, show that, in the absence of wind, a given speed of approach of the source raises the apparent pitch more than the same speed of approach of the receipient. Art. 100. Case IV.

1921.

1. Defiine the following :—amplitude, phase and period (Art 25) Show that the motion of a simple pendulum is approximately simple harmonic. (Page 184).

2. What do you mean by coefficient of volume elasticity? Prove that the coefficient of volume elasticity of a perfect gas under constant temperature is equal to the pressure. Arts 12 and 16.

3. Mention the different factors or sources of error that influence the velocity of sound as determined by open air observations, and explain how they may be corrected to obtain an absolute value. Art. 113

4. Give a general explanation of the manner in which Lissajou's figures may be observed and produced and how they may be practically utilised in acoustical determinations. Art 34

5. State how the presence of overtones in the sound of (*a*) a closed organ pipe, and (*b*) a bell may be observed and their pitches determined. Explain clearly the difference in the nature of the tone in the two cases. Arts. 68 and 95.

6. State the wave theory of light and deduce the laws of refraction of light on that theory. Arts 58, 63.

1922.

1. Distinguish between oscillatory motion and periodic motion. (Art. 17). Find an expression for the displacement at any instant of a particle executing simple harmonic motion of known period and amplitude. Explain how the time-displacement curve can be graphically drawn. Arts 25 and 27.

2. Briefly explain the theory and the method of finding the velocity of sound in air by the resonance column. Arts 68 and 112.

Calculate the velocity of sound in dry air at 0°*C* and 760 mm. pressure from the following data, supposing the density of moist air inside the tube to be ·0012 gram.

Length for the column of air for the first maximum, 33 cm.

Length for the second maximum, 181·6 cm.

Temperature of air inside the tube 30°*C*

Barometric height (corrected) 760 mm.

Frequency of the fork used, 256. (Arts. 47 and 112).

3. Define pitch and quality of a musical note. (Arts 11.) Explain how they are represented in the wave curve. (Figs 8 and 9). Explain graphically or methematically the formation of beats when two musical notes of nearly equal pitch are sounded together. Art. 99.

4. Explain the method of finding the absolute frequency of a tuning fork by the sonometer. What are the different sources of error, and how may these errors be minimised. Art.

1923.

1. Define the following—wave length, period, amplitude and phase. (Arts. 25 and 39) Show that the motion of the bob of a simple pendulum is approximately simple harmonic. (Page 184)

2. Give a general defination of the term elasticity, and prove that in the case of a gas, the elasticity at constant temperature is numerically equal to the pressure Arts. 12 and 16

3. Find an expression for the velocity of propagation of longitudinal vibrations in a solid rod, Arts. 44 and 89

Describe some method of finding the coefficient of longitudinal elasticity by an acoustical experiment. Art 120.

4. Describe any one form of Melde's experiment, and indicate how the laws of vibration of a stretched string can be verified with Melde's apparatus. Art. 78.

5. Clearly distinguish between overtones and harmonics. Give a few instances of overtones that are not harmonics.

How can you detect the presence of harmonics in a musical note, and what are their functions? Arts. 85 and 86

1924.

1. Define stress, strain and coefficient of Elasticity. Prove that, in the case of a gas, the elasticity at constant temperature is numerically equal to the pressure. Arts. 12 and 16

2. Find an expression for the velocity of sound in air from the theory of dimensions assuming that it depends on the pressure and the density*. Is the formula so obtained verified experimentally? Give reasons for your answer. Art. 46.

Calculate the velocity of sound in air at 0°C and pressure 760 mm. of mercury, given that the corresponding density of air is ·001293, $g=981$, the specific gravity of mercury is 13 6. Art. 47.

*Elasticity of a gas at constant temperature is equal to its pressure. As shown it Art. 16, elasticity is expressed is units of force per unit area.

Let L, M, T be the fundamental units of length, mass and time.

Then velocity $= L/T$, and acceleration $=$ rate of change of velocity with time $= L/T^2$

Force $=$ mass $\times$ acceleration $= \frac{ML}{T^2}$; Surface $= L^2$

Density $=$ mass per unit volume $= \frac{M}{L^3}$

Elasticity $= \frac{ML}{T^2} \div L^2 = \frac{M}{LT^2}$

Thus we have

$\frac{\text{Pressure}}{\text{Density}} = \frac{M}{LT^2} \div \frac{M}{L^3} = \frac{L^2}{T^2} = \text{velocity}^2$

$\therefore \quad V = \sqrt{\frac{P}{\rho}}$; where P = pressure of the gas, and ρ = its density.

3. Clearly explain the meaning of the expression "Velocity of propagation of a longitudinal wave." Represent graphically the state of disturbance at any time at any place during the propagation of a wave. Describe the mechanical process involved. Art. 9

Explain the formation of nodes and antinodes in an open organ pipe emitting its fundamental and first overtone. Art. 68.

4. Describe the method of compounding two vibrations at right angles to each other. Work out the case in which the phase difference is half of the period. What is the resultant effect of two vibrations in the same direction. Investigate all the different cases that arise. Arts. 31 and 32.

5. Compare the action of the phonograph and the telephone, and explain the characteristics of the vocal organ in man. Arts. 90, 91, 96.

1925.

1. Give Newton's formula for the velocity of sound in air. In what respect was it defective, and what correction was it found necessary to make that it might give accurate results. Art. 46

2. Explain the production of nodes and loops in the case of strings. Art. 66.

3. How has it been concluded that a sounding body, *e.g.* (*a*) a tuning-fork, (*b*) the air column in an organ-pipe is in vibratory motion ? Arts 35, 69.

To what pysical characteristics do the loudness and pitch of a musical note correspond ? Art. 11, 84.

4. What is wave-theory of light ? How would you explain the phenomena of reflection and refraction of light on the wave theory ? How does a wave of light differ from a wave of sound ? What is the nature of the vibration constituting light ? Art. 58, 63.

1926.

1. What do you mean by simple harmonic motions ? What is the nature of the force that will make a particle move in simple hormonic motion ? What reasons are there for believing that the motions that give rise to sound and light are simple harmonic ? Art. 19 *

* The wave theory is based on the simple harmonic motion of the particles of the medium through which the waves are propagated. This motion is transferred from particle to particle of the medium during wave propagation. The polarisation phenomenon of light leads us to infer that this motion of the particles is at right angles to the direction of wave propagation.

2. Explain the terms—pitch, interval, overtone, and timbre. Arts. 11, 84.

How would you experimentally demonstrate the existence of overtones in (*a*) a stretched string vibrating transversely, and in (*b*) a closed organ pipe ? Arts. 85, 86.

3. Give some examples of shadows in sound.

Explain clearly why sound shadows are generally not so well-marked as those of light. Arts. 101, 102.

4. Explain Doppler's principle.

A man standing by a railway notices the change of pitch of the note due to the whistle of an engine as it passes by him. If the frequency of the whistle be 256 vibrations per second and the velocity of the engine be $\frac{1}{20}$th of that of sound, what will be the frequencies of the notes heard by the man before and after the engine passes him? Art. 100, Ques. 1 page 139.

1927.

1. A particle P moves with a constant speed on a circle of radius 10 cm. making 3 complete revolutions per second. Discuss the motion of the projection Q of the particle P on a diameter of the circle, and find an expresssion for the velocity of Q at any instant. Calculate the position and the velocity of Q at the end of the first, second and third seconds. Arts. 23.

2. Explain, with the help of neat diagrams, the mode of vibrations of an open and of a closed organ pipe.

What will happen to the pitch and wave-length of the sound emitted if the density of the air is increased by 10 per cent. of its original value? Art. 68.

3. Explain how the pitch of the sound alters with the motion of the source or the observer.

Deduce a formula connecting the pitch of the note heard with the velocities of the source and the observer. Art. 100

4. Describe some experimental method by means of which wave-forms of musical sound may be studied. Arts. 34, 35.

1928.

1. Explain the terms 'periodic motion' and 'simple harmonic motion.'

Find graphically the resultant of two simple harmonic motions at right angles to each other, which have the same period and amplitude with a phase difference of $\frac{\pi}{4}$. Arts. 17, 32.

2. What do you understand by the terms—stress, strain, and coefficiept of elasticity? Art. 12.

3. State the law connecting the velocity of sound through a gas with its temperature and pressure.

If the velocity of sound through hydrogen at 0°C is 4,200 feet per second, what will be the velocity of sound (at the same temperature) through a mixture of two parts by volume of hydrogen to one of oxygen? (The density of oxygen is sixteen times that of hydrogen). Art. 47

$V=\sqrt{\frac{\lambda P}{\rho}}$ Let ρ = density of the mixture.

ρ_H = density of hydrogen.

ρ_0 = density of oxygen $=16\rho_H$

$2V\rho_H+V\rho_0=18V\rho_H=3V\rho$ $\therefore$ $\rho=6\rho_H$. Assuming that P is the same in both cases and the difference in the values of λ in the two cases negligible, we have

$$\frac{\text{velocity in hydrogen}}{\text{velocity in the mixture}}=\sqrt{\frac{16\rho_H}{\rho_H}}=4.$$

4. What is an echo? Why is a succession of echoes sometimes observed?

You staft walking away from a high wall clapping your hands once every second. How far must you go from the wall before you hear the echo of one clap simultaneously with the next clap?

(Speed of sound in air is 120 ft. per sec.) $\frac{2d}{V}=1$ $\therefore d=\frac{1,120}{2}=560$. ft.

5. Give carefully drawn diagrams showing the position of the nodes in an organ pipe, open at one end and closed at the other, for the fundamental vibration and the first three harmonics.

State the relations which exist between the lengths of closed and open organ pipes and the pitch of the notes emitted by them.

Art. 68.

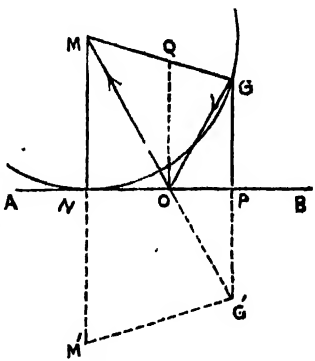
1929

1. What is meant by wave motion ? Describe the nature of vibrations which take place in ordinary water waves, in sound and light waves. What is meant by stationary waves ? Illustrate your answer by the vibrations of a string at one or both ends. Arts, 36, 8, 9, 102.

2. What is meant by elasticity of a body ? How is it measured ? Explain the terms you use. Prove that in the case of a gas the coefficient of elasticity at constant temparature is numerically equal to the pressure. Art. 12, 15, 16.

3. Explain the production of echoes. A gun is fired on the sea-shore in front of a line of cliffs. A man standing 300 ft. away from the gun and equidistant from the cliffs notices that the echo takes twice as long to reach him as does the direct report. Find by calculation or graphically the distance of the gun from the cliffs (velocity of sound 1,100 ft. per second) Art. 52

Ans. Let AB represent the line of cliffs, M the position of the man, and G the position of the gun. MN, the perpendicular distance of M from $AB=300$ ft. MG also is 300 ft. Therefore G and N lie on the circle with M as centre and MN as radius. Drop GP perpendicular on AB and produce it to G' making $PG'=GP$. Produce MN to NM' making $NM'=MN$. Then G' and M' are the images of G and M due rflection at AB. A ray GO incident on AB at O is reflected along OM making the ∠ of incidence $GOQ=$ ∠ of reflection QOM, OQ being the normal to AB at O.



Since the echo takes twice as long to reach M as does the direct report, we have

$GO+OM=2\ GM=2\times 300=600$ ft.

But $GO=OG'$ $\therefore$ $G'M=GO+OM=600$ ft.

Again $MM'=2\ MN=2\times 300=600$ ft.

$M'G=MG$ (by symmetry) $=300$ ft.

Let $GO=l$, and $GP=x$

To find x

$\angle GOP=\angle NOM$

$$\therefore \quad \text{Sin } GOP=\frac{GP}{GO}=\frac{x}{l}=\sin NOM=\cos NMO$$

$$=\frac{MN}{OM}=\frac{300}{600-l}$$

Again in the triangle $M'MG'$

$$\cos M'MG'=\frac{600^2+600^2-300^2}{2\times 600\times 600}=\frac{7}{8}$$

$$\frac{300}{600-l}=\frac{300+x}{600}=\frac{7}{8} \text{ or } x=225 \text{ ft.}$$

4. Explain and illustrate by diagrams the occurance of beats when two forks of nearly the same pitch are sounded together. How could you determine which fork is vibrating faster ? Art. 99

Ans. Load the prongs of one fork with small pieces of wax and see whether the frequency of the beats increases or decreases. If the frequency of the beats increases then this fork is of the lower frequency, because loading the fork lowers its own frequency. If the frequency of the beats decreases then this fork is of the higher frequency of the two.

5. The pitch of a musical note is altered if the source or the observer move relatively to one another. Explain clearly the reason of this phenomenon. Deduce an expression showing the relation between the alteration of pitch and the velocities of the source, the observer and the medium. Art. 100

1930

1. Show that the velocity of propagation of sound waves in a

gas is given by the expression $\sqrt{\frac{E}{\rho}}$, where E is the elasticity and ρ is the density of the gas. What expression will you use for the value of E? Give your reasons for choosing this particular value? Arts 44—46.

2. Give the theory of the formation of stationary waves in air and in other elastic media, and illustrate your answer by considering the different types of stationary waves with which you are familiar in acoustics. Arts. 66—69.

3. Write short notes on the following :—(*a*) interference of sound waves, (*b*) beats, and (*c*) combination tone. How can the existence of these phenomena are shown. Arts. 98, 99, 125

1931

1. Explain with diagrams the nature of the vibrations of a tuning fork. What special features make it valuable instrument in the scientifiic study of sound? Art. 92.

2. What is it that enables us to distinguish the sounds of the different vowels when we hear them, and how do the differences arise? Art. 90.

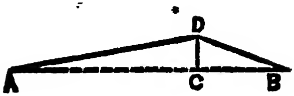
3. Explain why sound travels faster and is heard at greater distances in liquids and solids than in gases. Arts. 90, 117.

1932

1. Two simple harmonic oscillations of equal frequency and amplitude but in perpendicular directions are compounded. Find the resulting motion graphically. Art. 32

2. Prove that the frequency of vibrations of istretched string is equal to $\frac{1}{2l}\sqrt{\frac{T}{m}}$ where T is the tension, l the length, and m the mass per unit length of the string. Explain what harmonies wil

be present and what absent when the string is struck at the middle point.



When a point C of a string AB is pulled aside to D, the form taken up by it is that of two straight lines meeting at an obtnse angle at D. In the figure, CB is one third of AB. From Fourier's theorem it follows that the angular form ADB mignt be formed by adding ordinates of harmonic curves of which the length of the string contains an exact number of single bends. Of the infinite number of such curves only those are not required which cut the axis at C at which the string is pulled aside. CB being one third of AB, curve ADB might be formed by adding ordinates of harmonic curves, which have 1, 2, 4, 5, 7, 8, etc. of halfs waves in the length AB, those which have any multiple of 3 halfwaves being ommited. Thus if n be the frequency of the fundamental, the vibrations of frequencies n, $2n$, $4n$, $5n$, $7n$, $8n$, etc. are likely to be present in the note given out by the string, harmonics of frequencies $3n$, $6n$, $9x$ etc. being absent.

Similarly if $\frac{CB}{AB}=\frac{a}{b}$, $\frac{a}{b}$ being a fraction in its lowest term, then any harmonic curve which has mb single bends in the length AB, cuts the axis at C, where m is a whole number. The components of vibration of the string, therefore, are those whose frequencies are all multiples of n which are not multiples of bn.

Again, if the string be touched at some point with a light object, then all modes of vibrations of it except those which have a node at this point are stopped. Hence when a string is plucked at a point $\frac{a}{b}$ of its length and damped at a point $\frac{c}{d}$ of its length from one end, then the harmonic components of the vibrations will

be those whose frequencies are all multiples of dn but are not multiples of bn.

*In the question under consideration A being the middle point of AB, harmonics of frequencies $3n$, $5n$, $7n$ etc. only will be present along with the fundamental, and those of frequencies $4n$, $6n$, $8n$ etc. only will be absent.

3. Explain what you understand by the musical interval between two notes. What intervals are used in the diatonic major scale, what is temperament, and why is the tempered scale used in keyed instruments? Arts. 121, 122, 123.

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